1. Let $X$ be a discrete random variable that takes the values 1 with probability $p$ and $-1$ with probability $1 - p$. Let $Y$ be a continuous random variable independent of $X$ with the Laplacian (two-sided exponential) distribution
\[
f_Y(y) = \frac{\lambda}{2}e^{-\lambda|y|},
\]
and let $Z = X + Y$. Find $P(X = 1 \mid Z = z)$. Check that the expression obtained makes sense for $p \to 0^+$, $p \to 1^-$, $\lambda \to 0^+$, and $\lambda \to \infty$.

2. Let $Q$ be a continuous random variable with PDF
\[
f_Q(q) = \begin{cases} 
6q(1 - q), & \text{if } 0 \leq q \leq 1, \\
0, & \text{otherwise}.
\end{cases}
\]
This $Q$ represents the probability of success of a Bernoulli random variable $X$, i.e.,
\[
P(X = 1 \mid Q = q) = q.
\]
Find $f_{Q \mid X}(q \mid x)$ for $x \in \{0, 1\}$ and all $q$.

3. Let $X$ have the normal distribution with mean 0 and variance 1, i.e.,
\[
f_X(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}.
\]
Also, let $Y = g(X)$ where
\[
g(t) = \begin{cases} 
-t, & \text{for } t \leq 0; \\
\sqrt{t}, & \text{for } t > 0,
\end{cases}
\]
as shown to the right.
Find the probability density function of $Y$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{graph.png}
\caption{Graph of $g(t)$}
\end{figure}