6.041 Probabilistic Systems Analysis
6.431 Applied Probability

- Staff:
  - Lecturer: John Tsitsiklis
- Pick up and read course information handout
- Turn in recitation and tutorial scheduling form
  (last sheet of course information handout)
- Pick up copy of slides

Coursework
- Quiz 1 (October 12, 12:05-12:55pm) 17%
- Quiz 2 (November 2, 7:30-9:30pm) 30%
- Final exam (scheduled by registrar) 40%
- Weekly homework (best 9 of 10) 10%
- Attendance/participation/enthusiasm in recitations/tutorials 3%
- Collaboration policy described in course info handout
- Text: Introduction to Probability, 2nd Edition,
  Read the text!

LECTURE 1

- Readings: Sections 1.1, 1.2

Lecture outline
- Probability as a mathematical framework
  for reasoning about uncertainty
- Probabilistic models
  - sample space
  - probability law
- Axioms of probability
- Simple examples

Sample space $\Omega$
- “List” (set) of possible outcomes
- List must be:
  - Mutually exclusive
  - Collectively exhaustive
- Art: to be at the “right” granularity
Sample space: Discrete example

- Two rolls of a tetrahedral die
  - Sample space vs. sequential description

\[
\Omega = \{(x, y) \mid 0 \leq x, y \leq 1\}
\]

Sample space: Continuous example

\[
\Omega = \{(x, y) \mid 0 \leq x, y \leq 1\}
\]

Probability axioms

- **Event**: a subset of the sample space
- Probability is assigned to events

**Axioms:**
1. **Nonnegativity**: \( P(A) \geq 0 \)
2. **Normalization**: \( P(\Omega) = 1 \)
3. **Additivity**: If \( A \cap B = \emptyset \), then \( P(A \cup B) = P(A) + P(B) \)

\[
P(\{s_1, s_2, \ldots, s_k\}) = P(\{s_1\}) + \cdots + P(\{s_k\})
\]

= \( P(s_1) + \cdots + P(s_k) \)

- Axiom 3 needs strengthening
- Do weird sets have probabilities?

Probability law: Example with finite sample space

\[
\text{Let every possible outcome have probability 1/16}
\]

- \( P((X, Y) \text{ is } (1,1) \text{ or } (1,2)) = \)
- \( P(\{X = 1\}) = \)
- \( P(X + Y \text{ is odd}) = \)
- \( P(\min(X, Y) = 2) = \)
Discrete uniform law

- Let all outcomes be equally likely
- Then,
  \[ P(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}} \]
- Computing probabilities \( \equiv \) counting
- Defines fair coins, fair dice, well-shuffled decks

Continuous uniform law

- Two “random” numbers in \([0, 1]\).

- Uniform law: Probability = Area
  \[ P(X + Y \leq 1/2) = ? \]
  \[ P((X, Y) = (0.5, 0.3)) \]

Probability law: Ex. w/countably infinite sample space

- Sample space: \( \{1, 2, \ldots\} \)
  - We are given \( P(n) = 2^{-n}, n = 1, 2, \ldots \)
  - Find \( P(\text{outcome is even}) \)

\[ P(\{2, 4, 6, \ldots\}) = P(2) + P(4) + \cdots = \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots = \frac{1}{3} \]

- Countable additivity axiom (needed for this calculation):
  If \( A_1, A_2, \ldots \) are disjoint events, then:
  \[ P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots \]