Recitation 08 Answers  
March 09, 2006

1. (a) The marginal distributions are obtained by integrating the joint distribution along the X and Y axes and is shown in the following figure.

![Figure 1: Marginal probabilities $f_X(x)$ and $f_Y(y)$ obtained by integration along the y and x axes respectively](image)

The conditional PDFs are as shown in the figure below.

(b) X and Y are **NOT** independent since $f_{X|Y}(x|y) \neq f_X(x)f_Y(y)$. Also, from the figures we have $f_{X|Y}(x|y) \neq f_X(x)$.

(c)

$$f_{X,Y|A}(x,y) = \begin{cases} \frac{f_{X,Y}(x,y)}{P(A)} & (x,y) \in A \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{0.1}{0.1} & (x,y) \in A \\ 0 & \text{otherwise} \end{cases}$$

(d)

$$\mathbb{E}[X|Y = y] = \begin{cases} 0 & -2.0 \leq y \leq -1.0 \\ \frac{1}{3} & -1.0 \leq y \leq 1.0 \\ 0 & 1.0 \leq y \leq 2.0 \end{cases}$$
Figure 2: Conditional Probabilities
The conditional variance $\text{var}(X|Y = y)$ is given by

$$\text{var}(X|Y = y) = \begin{cases} \frac{4}{1600} & -2.0 \leq y \leq -1.0 \\ \frac{1}{4000} & -1.0 \leq y \leq 1.0 \\ \frac{4}{1600} & 1.0 \leq y \leq 2.0 \end{cases}$$

2. (a) We have $a = 1/800$, so that

$$f_{XY}(x, y) = \begin{cases} \frac{1}{1600} & 0 \leq x \leq 40 \text{ and } 0 \leq y \leq 2x \\ 0, & \text{otherwise.} \end{cases}$$

(b) $\mathbb{P}(Y > X) = 1/2$

(c) Let $Z = Y - X$. We have

$$f_Z(z) = \begin{cases} \frac{1}{1600}z + \frac{1}{40}, & -40 \leq z \leq 0, \\ -\frac{1}{1600}z + \frac{1}{40}, & 0 \leq z \leq 40, \\ 0, & \text{otherwise.} \end{cases}$$

$\mathbb{E}[Z] = 0.$