1. Suppose $X$ is uniformly distributed between $a$ and $b$.

   a) Find the transform of $X$.

   b) Use the transform in (a) to find the mean and the variance of $X$.

2. A three sided die is described by the following probabilities:

   \[ P(X = 1) = \frac{1}{2}, \quad P(X = 2) = \frac{1}{4}, \quad P(X = 3) = \frac{1}{4}. \]

   a) Find the transform of the above random variable.

   b) Use the transform to find the first three moments, $E[X], E[X^2], E[X^3]$.

   c) Check your answers in (b) by computing the moments directly.

3. Suppose a nonnegative discrete random variable has one of the following two expressions as its transform:

   (i) \[ M_X(s) = e^{2(e^s - 1)} \]

   (ii) \[ M_X(s) = e^{2(e^s - 1)} \]

   (a) Explain why one of the two could not possibly be its transform, and indicate which one is the true transform.

   (b) Find $P(X = 0)$.