Tutorial 05 Answer
March 16-17, 2006

1. (a) \[ f_Y(y) = 2y, \text{ for } 0 \leq y \leq 1. \]
   
   (b) \[ f_Y(y) = e^{-y}, \text{ for } 0 \leq y < \infty. \]

2. (a) \[ f_Y(y) = \frac{1}{\pi \sqrt{1-y^2}}, \text{ for } -1 < y < 1. \]
   
   (b) \[ f_Y(y) = \frac{1}{\pi} \cdot \frac{1}{1+y^2}, \text{ for } -\infty < y < \infty. \]

   A random variable with the above density is called a Cauchy random variable.

3. Optional
   
   (a) \[ f_Y(y) = \frac{1}{2\sqrt{y}} \cdot f_X(\sqrt{y}) + \frac{1}{2\sqrt{y}} \cdot f_X(-\sqrt{y}) \]
   \[ = \frac{1}{\sqrt{y}} \cdot f_X(\sqrt{y}), \text{ for } 0 \leq y < \infty. \]
   
   (b) \[ f_Y(y) = \frac{1}{y} f_X(\ln y), \text{ for } 0 \leq y < \infty. \]

   Note that \( f_X \) is the standard normal density in both (a) and (b).

4. (a) \[ f_{V,W}(v, w) = \frac{\log(1/v)}{2\sqrt{w}}, \quad 0 \leq v, w \leq 1 \]
   
   (b) \[ P(XY \leq Z^2) = P(V \leq W) = \int_0^1 \int_0^w \frac{\log(1/v)}{2\sqrt{w}} dv dw = \int_0^1 \left[ \frac{v(1 - \log v)}{2\sqrt{w}} \right]_{v=0}^w dw \]
   \[ = \int_0^1 \frac{\sqrt{w}(1 - \log w)}{2} dw = \left[ \frac{w^{3/2}}{3} \left( \frac{5}{3} - \log w \right) \right]_{w=0}^1 = \frac{5}{9} \]

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