Recitation 1: Solutions
September 9, 2010

1. Since the events $A \cap B^c$ and $A^c \cap B$ are disjoint, we have, using the additivity axiom,

$$P((A \cap B^c) \cup (A^c \cap B)) = P(A \cap B^c) + P(A^c \cap B).$$

Since $A = (A \cap B) \cup (A \cap B^c)$ is the union of two disjoint sets, we have, again by the additivity axiom,

$$P(A) = P(A \cap B) + P(A \cap B^c),$$

so that

$$P(A \cap B^c) = P(A) - P(A \cap B).$$

Similarly,

$$P(B \cap A^c) = P(B) - P(A \cap B).$$

Therefore,

$$P(A \cap B^c) + P(A^c \cap B) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B).$$

2. Let

$A :$ The event that the randomly selected student is a genius.

$B :$ The event that the randomly selected student loves chocolate.

From the properties of probability laws proved in lecture, we have

$$1 = P(A \cup B) + P((A \cup B)^c)$$

$$= P(A) + P(B) - P(A \cap B) + P(A^c \cap B^c)$$

$$= 0.6 + 0.7 - 0.4 + P(A^c \cap B^c)$$

$$= 0.9 + P(A^c \cap B^c).$$

Therefore

$$P(A \text{ randomly selected student is neither a genius nor a chocolate lover})$$

$$= P(A^c \cap B^c) = 1 - 0.9 = 0.1.$$

3. Let $c$ denote the probability of a single odd face. Then the probability of a single even face is $2c$, and by adding the probabilities of the 3 odd faces and the 3 even faces, we get $9c = 1$. Thus, $c = 1/9$. The desired probability is

$$P(\{1, 2, 3\}) = P(\{1\}) + P(\{2\}) + P(\{3\}) = c + 2c + c = 4c = 4/9.$$

4. See the textbook, Example 1.5, page 13.

G1†. See the textbook, Problem 1.13, page 56.