Recitation 5 Solutions  
September 23, 2010

1. (a) See derivation in textbook pp. 84-85.  
   (b) See derivation in textbook p. 86.  
   (c) See derivation in textbook p. 87.

2. (a) $X$ is a Binomial random variable with $n = 10$, $p = 0.2$. Therefore,

   \[ p_X(k) = \binom{10}{k} 0.2^k 0.8^{10-k}, \quad \text{for } k = 0, \ldots, 10 \]

   and $p_X(k) = 0$ otherwise.

   \[
   \begin{array}{c|c|c}
   \text{number of hits} & \text{probability} \\
   \hline
   0 & 0.1074 \\
   1 & \ldots \\
   10 & \ldots \\
   \end{array}
   \]

   (b) $P(\text{No hits}) = p_X(0) = (0.8)^{10} = \boxed{0.1074}$

   (c) $P(\text{More hits than misses}) = \sum_{k=6}^{10} p_X(k) = \sum_{k=6}^{10} \binom{10}{k} 0.2^k 0.8^{10-k} = \boxed{0.0064}$

   (d) Since $X$ is a Binomial random variable,

   \[ E[X] = 10 \cdot 0.2 = 2 \quad \text{var}(X) = 10 \cdot 0.2 \cdot 0.8 = 1.6 \]

   (e) $Y = 2X - 3$, and therefore

   \[ E[Y] = 2E[X] - 3 = 1 \quad \text{var}(Y) = 4\text{var}(X) = 6.4 \]

   (f) $Z = X^2$, and therefore

   \[ E[Z] = E[X^2] = (E[X])^2 + \text{var}(X) = 5.6 \]

3. (a) We expect $E[X]$ to be higher than $E[Y]$ since if we choose the student, we are more likely to pick a bus with more students.

   (b) To solve this problem formally, we first compute the PMF of each random variable and then compute their expectations.

   \[ p_X(x) = \begin{cases} 
   40/148 & x = 40 \\
   33/148 & x = 33 \\
   25/148 & x = 25 \\
   50/148 & x = 50 \\
   0 & \text{otherwise}. 
   \end{cases} \]
and \( E[X] = 40 \frac{40}{148} + 33 \frac{33}{148} + 25 \frac{25}{148} + 50 \frac{50}{148} = 39.28 \)

\[
p_Y(y) = \begin{cases} 
1/4 & y = 40, 33, 25, 50 \\
0 & \text{otherwise}.
\end{cases}
\]

and \( E[Y] = 40 \frac{1}{4} + 33 \frac{1}{4} + 25 \frac{1}{4} + 50 \frac{1}{4} = 37 \)

Clearly, \( E[X] > E[Y] \).

4. The expected value of the gain for a single game is infinite since if \( X \) is your gain, then

\[
\sum_{k=1}^{\infty} 2^k \cdot 2^{-k} = \sum_{k=1}^{\infty} 1 = \infty
\]

Thus if you are faced with the choice of playing for given fee \( f \) or not playing at all, and your objective is to make the choice that maximizes your expected net gain, you would be willing to pay any value of \( f \). However, this is in strong disagreement with the behavior of individuals. In fact experiments have shown that most people are willing to pay only about $20 to $30 to play the game. The discrepancy is due to a presumption that the amount one is willing to pay is determined by the expected gain. However, expected gain does not take into account a person’s attitude towards risk taking.

Below are histograms showing the payout results for various numbers of simulations of this game:
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