

Tutorial 5
October 14/15, 2010

1. Let Q be a random variable which is uniformly distributed between 0 and 1. On any given day, a particular machine is functional with probability Q . Furthermore, given the value of Q , the status of the machine on different days is independent.

(a) Find the probability that the machine is functional on a particular day.

(b) We are told that the machine was functional on m out of the last n days. Find the conditional PDF of Q . You may use the identity

$$\int_0^1 p^k (1-p)^{n-k} dp = \frac{k!(n-k)!}{(n+1)!}$$

2. Let X be a random variable with PDF f_X . Find the PDF of the random variable $Y = |X|$

(a) when $f_X(x) = \begin{cases} 1/3, & \text{if } -2 < x \leq 1, \\ 0, & \text{otherwise;} \end{cases}$

(b) when $f_X(x) = \begin{cases} 2e^{-2x}, & \text{if } x > 0, \\ 0, & \text{otherwise;} \end{cases}$

(c) for general $f_X(x)$.

3. An ambulance travels back and forth, at a constant specific speed v , along a road of length ℓ . We may model the location of the ambulance at any moment in time to be uniformly distributed over the interval $(0, \ell)$. Also at any moment in time, an accident (not involving the ambulance itself) occurs at a point uniformly distributed on the road; that is, the accident's distance from one of the fixed ends of the road is also uniformly distributed over the interval $(0, \ell)$. Assume the location of the accident and the location of the ambulance are independent.

Supposing the ambulance is capable of *immediate* U-turns, compute the CDF and PDF of the ambulance's travel time T to the location of the accident.

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