Problem 27.* We toss $n$ times a biased coin whose probability of heads, denoted by $q$, is the value of a random variable $Q$ with given mean $\mu$ and positive variance $\sigma^2$. Let $X_i$ be a Bernoulli random variable that models the outcome of the $i$th toss (i.e., $X_i = 1$ if the $i$th toss is a head). We assume that $X_1, \ldots, X_n$ are conditionally independent, given $Q = q$. Let $X$ be the number of heads obtained in the $n$ tosses.

(a) Use the law of iterated expectations to find $E[X_i]$ and $E[X]$.

(b) Find $\text{cov}(X_i, X_j)$. Are $X_1, \ldots, X_n$ independent?

(c) Use the law of total variance to find $\text{var}(X)$. Verify your answer using the covariance result of part (b).