LECTURE 13
The Bernoulli process

• **Readings:** Section 6.1

**Lecture outline**
• Definition of Bernoulli process
• Random processes
• Basic properties of Bernoulli process
• Distribution of interarrival times
• The time of the kth success
• Merging and splitting

---

The Bernoulli process

• A sequence of independent Bernoulli trials

• At each trial, i:
  – $P(\text{success}) = P(X_i = 1) = p$
  – $P(\text{failure}) = P(X_i = 0) = 1 - p$

• Examples:
  – Sequence of lottery wins/losses
  – Sequence of ups and downs of the Dow Jones
  – Arrivals (each second) to a bank
  – Arrivals (at each time slot) to server

---

Random processes

• First view:
  sequence of random variables $X_1, X_2, \ldots$

• $E[X_t] =$

• $\text{Var}(X_t) =$

• Second view:
  what is the right sample space?

• $P(X_t = 1 \text{ for all } t) =$

• Random processes we will study:
  – Bernoulli process
    (memoryless, discrete time)
  – Poisson process
    (memoryless, continuous time)
  – Markov chains
    (with memory/dependence across time)

---

Number of successes $S$ in $n$ time slots

• $P(S = k) =$

• $E[S] =$

• $\text{Var}(S) =$
Interarrival times

- $T_1$: number of trials until first success
  - $P(T_1 = t) =$
  - Memoryless property
  - $E[T_1] =$
  - $\text{Var}(T_1) =$

- If you buy a lottery ticket every day, what is the distribution of the length of the first string of losing days?

Time of the $k$th arrival

- Given that first arrival was at time $t$ i.e., $T_1 = t$:
  - additional time, $T_2$, until next arrival
  - has the same (geometric) distribution
  - independent of $T_1$

- $Y_k$: number of trials to $k$th success
  - $E[Y_k] =$
  - $\text{Var}(Y_k) =$
  - $P(Y_k = t) =$

Splitting of a Bernoulli Process

(Using independent coin flips)

yields Bernoulli processes

Merging of Indep. Bernoulli Processes

yields a Bernoulli process (collisions are counted as one arrival)