

Recitation 17
November 4, 2010

- Iwana Passe is taking a multiple-choice exam. You may assume that the number of questions is infinite. *Simultaneously, but independently*, her conscious and subconscious faculties are generating answers for her, each in a Poisson manner. (Her conscious and subconscious are always working on different questions.) Conscious responses are generated at the rate λ_c responses per minute. Subconscious responses are generated at the rate λ_s responses per minute. Assume $\lambda_c \neq \lambda_s$. Each conscious response is an independent Bernoulli trial with probability p_c of being correct. Similarly, each subconscious response is an independent Bernoulli trial with probability p_s of being correct. Iwana responds only once to each question, and you can assume that her time for recording these conscious and subconscious responses is negligible.
 - Determine $p_K(k)$, the probability mass function for the number of *conscious responses* Iwana makes in an interval of T minutes.
 - If we pick any question to which Iwana has responded, what is the probability that her answer to that question:
 - Represents a conscious response
 - Represents a conscious correct response
 - If we pick an interval of T minutes, what is the probability that in that interval Iwana will make exactly r conscious responses *and* s subconscious responses?
 - Determine the probability density function for random variable X , where X is the time from the start of the exam until Iwana makes her first conscious response which is preceded by at least one subconscious response.
- Shem, a local policeman, drives from intersection to intersection in times that are independent and all exponentially distributed with parameter λ . At each intersection he observes (and reports) a car accident with probability p . (This activity does not slow his driving at all.) Independently of all else, Shem receives extremely brief radio calls in a Poisson manner with an average rate of μ calls per hour.
 - Determine the PMF for N , the number of intersections Shem visits up to and including the one where he reports his first accident.
 - Determine the PDF for Q , the length of time Shem drives between reporting accidents.
 - What is the PMF for M , the number of accidents which Shem reports in two hours?
 - What is the PMF for K , the number of accidents Shem reports between his receipt of two successive radio calls?
 - We observe Shem at a random instant long after his shift has begun. Let W be the total time from Shem's last radio call until his next radio call. What is the PDF of W ?
- Problem 6.27, page 337 in the textbook. **Random incidence in an Erlang arrival process.** Consider an arrival process in which the interarrival times are independent Erlang random variables of order 2, with mean $2/\lambda$. Assume that the arrival process has been ongoing for a very long time. An external observer arrives at a given time t . Find the PDF of the length of the interarrival interval that contains t .

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