Mathematics for Computer Science

MIT 6.042J/18.062J

Drawing

Planar Graphs
Planar Graphs

A graph is *planar* if there is a way to *draw* it in the plane without edges crossing.
Planar Graphs

Maps are 2-connected planar graphs
Planar Graphs

Maps are 2-connected planar graphs

General connected planar graphs may have
Planar Graphs

Maps are 2-connected planar graphs

General connected planar graphs may have loops
Planar Graphs

Maps are 2-connected planar graphs

General connected planar graphs may have
dongles cross bars
A Planar Graph
A Planar Graph

with 3 *faces*
A Planar Graph

with 3 \textit{faces}
A Planar Graph

with 3 faces (wait! also the outer face)
A Planar Graph

with 4 faces (wait! also the outer face)
Drawing a Planar Graph

draw it edge by edge, starting with a single vertex
Drawing a Planar Graph
draw it edge by edge,
starting with a single vertex
Drawing a Planar Graph
and record faces while drawing

graph
Drawing a Planar Graph
and record faces while drawing graph.

faces.
Planar Graphs

and record faces while drawing

graph

faces
Planar Graphs

and record faces while drawing

graph

faces
Planar Graphs

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graph

faces
Planar Graphs

and record faces while drawing

graph

faces
Planar Graphs

and record faces while drawing

graph

faces

(the outer face)
Planar Graphs

and record faces while drawing

graph

faces

(the outer face)
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faces

(the outer face)
Planar Graphs

and record faces while drawing

graph

faces

(the outer face)
Recursive Definition of Faces

Precise rules defining the cycles that are the face boundaries of a Planar Drawing:
Recursive Definition of Faces

Start with a vertex
Recursive Definition of Faces

Start with a vertex

There is one face, whose boundary is the 0-length cycle consisting of this vertex.
Recursively adding an edge to a drawing

Two cases for connected graph:
1) Attach edge from vertex on a face to a new vertex.
2) Attach edge between nonadjacent vertices on a face.
Face Creation Rule 1

1) choose vertex, \( v \), on a face boundary
Face Creation Rule 1

1) choose vertex, $v$, on a face boundary
Face Creation Rule 1

1) choose vertex, \( v \), on a face boundary path \( x \)
Face Creation Rule 1

1) choose vertex, $v$, on a face boundary

face boundary $v x v$
Face Creation Rule 1

1) choose vertex, $v$, on a face boundary

Create new vertex, $w$, 
Face Creation Rule 1

1) choose vertex, $v$, on a face boundary

Create new vertex, $w$, and add edge $v--w$
Face Creation Rule 1

old face boundary $v$ $w$ $x$ $v$ $w$

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Face Creation Rule 1

new face boundary

path x
Face Creation Rule 1

path $x$

new face boundary
Face Creation Rule 1

new face boundary $vwvxv$
Face Creation Rules

nothing else changes

new face boundary vwxv
Recursive Face Creation Rule 2

2) choose vertices $v, w$ on a face boundary
Face Creation Rule 2

2) choose vertices \( v, w \) on a \textbf{face} boundary
Face Creation Rule 2

2) choose vertices $v, w$ on a face boundary

with $v, w$, not adjacent
Face Creation Rule 2

2) choose vertices $v$, $w$ on a face boundary

face boundary $vywxv$
Face Creation Rule 2

2) choose vertices \( v, w \) on a face boundary

and add edge \( v--w \)
Face Creation Rule 2

old face boundary $vywvxv$
Face Creation Rule 2

old face boundary $vyw\hat{w}xv$
Face Creation Rule 2

splits into 2 faces:
Face Creation Rule 2

splits into 2 faces:
Face Creation Rule 2

splits into 2 faces: $vwxv$
Face Creation Rule 2

splits into 2 faces: $vwxv$, $vywv$
Face Creation Rules

nothing else changes

splits into 2 faces: $vwxv, vywv$
Recursive Definition of Faces

Every connected planar drawing is obtained by starting with a single vertex, and repeatedly applying Rules 1 & 2.
Induction on Drawings

Properties of planar drawings like Euler’s formula can be proved by induction on the number of rule applications used to create a drawing.