Airline Gate Allocation
Given a set of airline flights needing gates at overlapping times, how many different gates do I need in order to accommodate them?

Model as a Graph
Needs gate at the same time

Airline Schedule
Flight Numbers

Model as a Graph
Color vertices so that adjacent vertices have different colors.

# colors = # gates needed

**Better: 3 Colors**

so 3 airline gates will do

**Final Exam Time Slots**

Two subjects conflict if a student is taking both. Assign conflicting subjects to different time slots, keeping exam period short.

**Model as a Graph**

4 time slots (best possible)

**More Conflicting Allocation Problems**

- # separate habitats to house different species of animals, some incompatible with others?
- # different frequencies for radio stations that interfere with each other?
- # different colors to color a map?
Map Coloring

Countries are the Vertices

Four Color Theorem
Any planar map is 4-colorable. False proof published 1850’s (was correct for 5 colors). Proof with computer calculations: 1970’s. Much improved: 1990’s

Chromatic Number
\[ \chi(G) = \text{Chromatic Number of } G \]
\[ := \text{minimum } \#\text{colors for } G \]

Trees are 2-colorable
Pick any vertex as “root.” If (unique) path from root to \( w \) is odd length: \( \text{even length: } w \)

Simple Cycles
\[ \chi(C_{\text{even}}) = 2 \]
\[ \chi(C_{\text{odd}}) = 3 \]
### Complete Graph $K_5$

$$\chi(K_n) = n$$

### The Wheel $W_n$

$$W_5$$

$$\chi(W_{\text{odd}}) = 4 \quad \chi(W_{\text{even}}) = 3$$

### Bounded Degree

If $\deg(v) \leq k$ for all vertices $v$ of $G$, then

$$\chi(G) \leq k + 1$$

A simple recursive coloring procedure achieves this.

### Arbitrary Graphs

2-colorable? -- easy to check
3-colorable? -- hard to check (even if planar)

$$\chi(G)?$$ -- harder still

### Team Problems

Problems 2 & 3