Probabilistic Diagnosis

99% accurate TB testing
A great-sounding diagnostic test for TB:

If you have TB, the test is guaranteed to detect it.

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A great-sounding diagnostic test for TB: if you have TB, the test is guaranteed to detect it. If you don't have TB, the test says so 99% of the time.
A great-sounding diagnostic test for TB: if you have TB the test is **guaranteed** to detect it. If you don’t have TB, the test says so 99% of the time. Your doctor gives you the test, and it says you have TB!

TB is a serious disease and the test is at least 99% accurate. How worried should you be? What is the probability that you actually have TB?

Do you have TB?

What is the probability that you have TB given that a 99% accurate says you do?

\[
\Pr[TB | +] = ?
\]

“+” for [test positive]

Pr[ + | not TB] = \frac{1}{100} = 0.01

false positive rate only 1%
Do you have TB?

\[ \Pr[TB | +] = \frac{\Pr[TB \text{ AND } +]}{\Pr[+]}, \]

\[ = 1 \]

\[ = \frac{\Pr[+|TB]}{\Pr[TB]} \]

You do or you don't

\[ \Pr[+] = \]

Total Probability Rule

\[ \Pr[+] = \Pr[+ | TB] \cdot \Pr[TB] \]

\[ + \Pr[+ | \text{not TB}] \cdot \Pr[\text{not TB}] \]

Total Probability Rule
You do or you don’t

\[
\Pr[+] = \Pr[+|TB] \cdot \Pr[TB]
\]
\[
+ \Pr[+|not \ TB] \cdot \Pr[not \ TB]
\]
\[
= 1 \cdot \Pr[TB]
\]
\[
+ \frac{1}{100} \cdot \Pr[not \ TB]
\]

Probability of Testing Positive

\[
\Pr[+] = \Pr[+|TB] \cdot \Pr[TB]
\]
\[
+ \Pr[+|not \ TB] \cdot \Pr[not \ TB]
\]
\[
= \frac{99}{100} \Pr[TB] + \frac{1}{100}
\]

Do you have TB?

\[
\Pr[TB|+] = \frac{\Pr[TB]}{\Pr[+]} = \frac{\frac{\Pr[TB]}{\frac{99}{100} \Pr[TB] + \frac{1}{100}}}{\Pr[+]} = \frac{\frac{\Pr[TB]}{\frac{99}{100} \Pr[TB] + \frac{1}{100}}}{\frac{99}{100} \Pr[TB] + \frac{1}{100}}
\]
Do you have TB?

\[ \Pr[TB | +] = \frac{\Pr[TB]}{\Pr[+]} = \frac{100\Pr[TB]}{99\Pr[TB] + 1} \]

What is \( \Pr[TB] \)?

11,000 TB cases reported

CDC got reports of 11,000 cases of TB in US in 2011. Will be lots of unreported. So estimate:

\[ \Pr[TB] \approx \frac{1}{10,000} \]

Unlikely you have TB

Because of relatively high false positive rate (1%) compared to TB rate (0.01%), chance of having TB remains small (1%)!
Unlikely you have TB

99% accurate test is not so good here.

A “more accurate” test

99% accurate test is not so good here. In fact, there’s a trivial test that is 99.99% accurate:

always say “No TB”

Bayes Rule

\[
\Pr[\text{TB} | +] = \frac{\Pr[+ | \text{TB}] \cdot \Pr[\text{TB}]}{\Pr[+]} \\
\Pr[B | A] = \frac{\Pr[A | B] \cdot \Pr[B]}{\Pr[A]}
\]

99% accuracy still useful

99% accurate test did increase your probability of TB 100 times.
99% accurate test did increase your probability of TB 100 times. If you only had 5M medicine doses for a population of 350M, whom should you medicate?

Medicate the 3.5M who test positive, and you’re likely to cure nearly all the cases.