Introduction to Random Variables

Bigger Number Game

Guess the Bigger Number

Team 1:
• Write two integers from 0 to 7 on two pieces of paper
• Show to Team 2 face down

Team 2:
• Expose one paper and look at number
• Either stick or switch to other number

Team 2 wins if gets larger number

Do you think one team has an advantage?

You might like to try playing the game a few times with some teammates before seeing the answers below.
Strategy for Team 2

- pick a paper to expose, giving each paper equal probability.
- if exposed number is “small” then switch, otherwise stick. That is switch if \( \leq \text{threshold } Z \) where \( Z \) is a random integer \( \in [0,7) \)

Analysis of Team 2 Strategy

Let low < high be the integers chosen by Team 1. There are three cases:

Case M: low \( \leq Z < \) high
Team 2 wins in this case, so
\[
\Pr[\text{Team 2 wins } | M] = 1
\]
and
\[
\Pr[M] \geq \frac{1}{7}
\]

Case H: high \( \leq Z \)
Team 2 will switch, so wins iff low card gets exposed
\[
\Pr[\text{Team 2 wins } | H] = \frac{1}{2}
\]
Analysis of Team 2 Strategy

Case L: \( Z < \text{low} \)
Team 2 will stick, so wins iff high card gets exposed
\[
\Pr[\text{Team 2 wins} \mid L] = \frac{1}{2}
\]

So \( \geq \frac{1}{7} \) of time, sure win.
Rest of time, win \( \frac{1}{2} \).

By Law of Total Probability

\[
\Pr[\text{Team 2 wins}] \geq \frac{1}{7} \cdot \frac{1}{2} + \frac{1}{7} \cdot \left( 1 - \frac{1}{7} \right) = \frac{4}{7}
\]
Analysis of Team 2 Strategy

So Team 2 has the advantage, no matter what Team 1 does!

Team 1 Strategy

...& Team 1 can play so

\[ \Pr[\text{Team 2 wins}] \leq \frac{4}{7} \]

no matter what

Optimal Strategy

\[ \Pr[\text{Team 2 wins}] = \frac{4}{7} \]

is optimal for both
Random Variables
Informally: an RV is a number produced by a random process:
• threshold variable $Z$
• number of exposed card
• number of larger card
• number of smaller card
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