


Mathematics for Computer Science
 MIT 6.042J/18.062J

Bounds on Deviation

Chebyshev Bound


 Albert R Meyer, May 10, 2013 chebyshev.1


Improving the Markov Bound


$$\Pr[|R - \mu| \geq x]$$


$$= \Pr[(R - \mu)^2 \geq x^2]$$

by Markov:

$$\leq \frac{E[(R - \mu)^2]}{x^2}$$


variance of R



 Albert R Meyer, May 10, 2013 chebyshev.2


Chebyshev Bound

$$\Pr[|R - \mu| \geq x] \leq \frac{\text{Var}[R]}{x^2}$$

$$\text{Var}[R] ::= E[(R - \mu)^2]$$



 Albert R Meyer, May 10, 2013 chebyshev.3



Variance of a Random Variable

$$\text{Var}[R] ::= E[(R - \mu)^2]$$

Variance is also called the

mean square error



 Albert R Meyer, May 10, 2013 chebyshev.4


 **Chebyshev Bound**

$$\Pr[|R - \mu| \geq x] \leq \frac{\text{Var}[R]}{x^2}$$

$$\sigma_R ::= \sqrt{\text{Var}[R]}$$

standard deviation

 Albert R Meyer, May 10, 2013 chebyshev.5


 **Standard Deviation of an RV**


Standard deviation is also called the

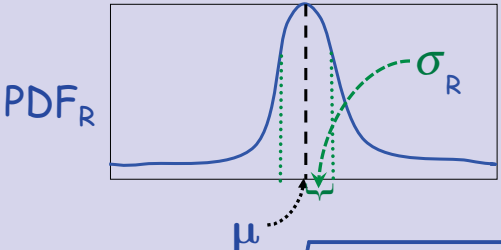
root mean square error

$$\sigma_R ::= \sqrt{\text{Var}[R]}$$

standard deviation


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
 **Standard Deviation of an RV**



PDF_R


$$\sigma_R ::= \sqrt{\text{Var}[R]}$$


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 **Chebyshev Bound**

$$\Pr[|R - \mu| \geq x] \leq \frac{\sigma_R^2}{x^2}$$


$$\sigma_R ::= \sqrt{\text{Var}[R]}$$

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Chebyshev Bound

$$\Pr[|R - \mu| \geq x] \leq \frac{\sigma_R^2}{x^2}$$


$$\sigma_R ::= \sqrt{\text{Var}[R]}$$


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Chebyshev Bound (Restated)

$$\Pr[|R - \mu| \geq c\sigma_R] \leq \frac{1}{c^2}$$

$$\sigma_R ::= \sqrt{\text{Var}[R]}$$


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Standard Deviation

$$\Pr[|R - \mu| \geq c\sigma_R] \leq \frac{1}{c^2}$$

R probably not many σ 's from μ :
 further than

σ	$\Pr \leq 1$
2σ	$\Pr \leq 1/4$
3σ	$\Pr \leq 1/9$
4σ	$\Pr \leq 1/16$



Albert R Meyer, May 10, 2013 chebyshev.11

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