Simple Graphs: Coloring

Flight Gates
flights need gates, but times overlap.
how many gates needed?

Airline Schedule
Conflicts Among 3 Flights
Needs gate at same time
Model all Conflicts with a Graph

Color the vertices

Color vertices so that adjacent vertices have different colors.

\[
\text{min \ # distinct colors needed} = \text{min \ # gates needed}
\]

Coloring the Vertices

Better coloring

assign gates:

4 colors
4 gates

3 colors
3 gates
Final Exams

Subjects conflict if student takes both, so need different time slots. How short an exam period?

Model as a Graph

Assign times:

4 time slots (best possible)

Map Coloring

Conflicting Allocation Problems

# Separate habitats to house different species of animals, some incompatible with others?
# Different frequencies for radio stations that interfere with each other?
# Different colors to color a map?
**Countries are the Vertices**

![Diagram of countries as vertices]

**Planar Four Coloring**

Any planar map is 4-colorable. 1850’s: false proof published (was correct for 5 colors). 1970’s: proof with computer. 1990’s: much improved.

**Chromatic Number**

\[ \chi(G) \]

\[ \text{min #colors for } G \text{ is chromatic number} \]

**Simple Cycles**

\[ \chi(C_{\text{even}}) = 2 \]

\[ \chi(C_{\text{odd}}) = 3 \]
Complete Graph $K_n$

$\chi(K_n) = n$

The Wheel $W_n$

$W_5$

$\chi(W_{\text{odd}}) = 4$

$\chi(W_{\text{even}}) = 3$

Bounded Degree

all degrees $\leq k$, implies

$\chi(G) \leq k+1$

very simple algorithm...

"Greedy" Coloring

...color vertices in any order.
next vertex gets a color different from its neighbors.
$\leq k$ neighbors, so
$k+1$ colors always work
coloring arbitrary graphs

2-colorable? --easy to check
3-colorable? --hard to check
(even if planar)

find $\chi(G)$? --theoretically
no harder than 3-color, but
harder in practice