A simple graph:

Definition:
A simple graph $G$ consists of
• a nonempty set, $V$, of vertices, and
• a set, $E$, of edges such that
each edge has two endpoints in $V$
**Vertex degree**

degree of a vertex is 
# of incident edges

\[ \text{deg}(\bullet) = 2 \]

**Vertex degree**

degree of a vertex is 
# of incident edges

\[ \text{deg}(\bullet) = 4 \]

**Impossible Graph**

Is there a graph with vertex degrees 2, 2, 1?

**NO!**

\[ \begin{array}{c}
\text{2} \\
\text{1} \\
\text{2}
\end{array} \]

orphaned edge

**Handshaking Lemma**

sum of degrees is 
twice # edges

\[ 2|E| = \sum_{v \in V} \text{deg}(v) \]

Proof: Each edge contributes 2 to the sum on the right
Handshaking Lemma
sum of degrees is twice # edges
\[ 2|E| = \sum_{v \in V} \deg(v) \]
2+2+1 = odd, so impossible

Sex in America: Men more Promiscuous?
Studies claim different %’s but agree that men average many more partners than women.
Graph theory shows this is nonsense

Sex Partner Graph

M

F

partners
### Counting pairs of partners

**avg degree(M)** := \( \sum_{m \in M} \frac{\text{deg}(m)}{|M|} \)

\[ \text{avg degree}(M) := \frac{\sum_{m \in M} \text{deg}(m)}{|M|} \]

**avg degree(F)** := \( \sum_{f \in F} \frac{\text{deg}(f)}{|F|} \)

\[ \text{avg degree}(F) := \frac{\sum_{f \in F} \text{deg}(f)}{|F|} \]

**avg degree(M)** := \( \frac{\sum_{m \in M} \text{deg}(m)}{|M|} \)

**avg degree(F)** := \( \frac{\sum_{f \in F} \text{deg}(f)}{|F|} \)

\[ \sum_{m \in M} \text{deg}(m) = |E| = \sum_{f \in F} \text{deg}(f) \]

### Average number of partners

\[ \text{avg - deg}(M) = 1.035 \cdot \text{avg - deg}(F) \]

**Averages differ solely by ratio of females to males.**

**No big difference**

**Nothing to do with promiscuity**
Why are surveys wrong?

Maybe people are *lying*:

- Males exaggerate?
- Females deny?

Maybe Males have partners outside the study