Equivalence Relations

two-way walks
walk from \( u \) to \( v \) and back from \( v \) to \( u \):
\( u \) and \( v \) are strongly connected.
\( u G^* v \) AND \( v G^* u \)

symmetry
relation \( R \) on set \( A \) is symmetric iff
\( a R b \) IMPLIES \( b R a \)

equivalence relations
transitive, symmetric & reflexive
Theorem: R is an equiv rel iff R is the strongly connected relation of some digraph.

equivalence relations

equivalence relations

equivalence relations

examples:

• = (equality)
• ≡ (mod n)
• same size
• same color

Graphical Properties of Relations

Reflexive

Asymmetric

Transitive

Symmetric

Representing Equivalences
Representing equivalences

For total function \( f : A \rightarrow B \) define relation \( \equiv_f \) on \( A \):
\[ a \equiv_f a' \iff f(a) = f(a') \]

Theorem:
Relation \( R \) on set \( A \) is an equiv. relation IFF
\( R \) is \( \equiv_f \) for some \( f : A \rightarrow B \)

representing \( \equiv \) \( (\mod n) \)
\[ \equiv (\mod n) \] is \( \equiv_f \) where
\[ f(k) ::= \text{rem}(k,n) \]

For partition \( \Pi \) of \( A \) define relation \( \equiv_\Pi \) on \( A \):
\[ a \equiv_\Pi a' \iff a, a' \text{ are in the same block of } \Pi \]
Representing equivalences

Theorem:
Relation $R$ on set $A$ is an equiv. relation IFF

$R$ is $\equiv_{\Pi}$

for some partition $\Pi$ of $A$