**Inclusion-Exclusion**

2 set proof

\[ |A \cup B| = |A| + |B| - |A \cap B| \]

**Inc-Exc from Sum Rule**

\[ A \cup B = A \cup (B - A) \]

proof: disjoint

by Sum Rule
Theorem 5.1: Inclusion-Exclusion (2 Sets)

\[ |A \cup B| = |A| + |B| - |A \cap B| \]

**Proof:**

\[ B = (B \cap A) \cup (B - A) \]

disjoint

Lemma 5.2: Inclusion-Exclusion (3 Sets)

\[ |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \]

**Proof:**

by Sum Rule

QED
Incl-Excl (n sets)

\[ A_1 \cup A_2 \cup \cdots \cup A_n = \sum_{\emptyset \neq S \subseteq \{1, 2, \ldots, n\}} (-1)^{|S|+1} \bigcap_{i \in S} A_i \]

Incl-Excl Formula: Proofs

by induction on \( n \) --uninformative

by binomial counting --next

by distributivity --also