Simple Graphs: Isomorphism

The Graph Abstraction

Same graph (different layouts)

All that matters are the connections: graphs with the same connections are isomorphic
**Isomorphism**

Two graphs are isomorphic when there is an edge-preserving matching of their vertices.

**Are these isomorphic?**

```
<table>
<thead>
<tr>
<th></th>
<th>Dog</th>
<th>Pig</th>
<th>Cat</th>
<th>Cow</th>
<th>Beef</th>
<th>Tuna</th>
<th>Hay</th>
<th>Corn</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(Dog)</td>
<td>Beef</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f(Cat)</td>
<td>Tuna</td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>
```

**Edges preserved?**

- **YES!**
Formal Def of Graph Isomorphism

$G_1$ isomorphic to $G_2$ means edge-preserving vertex matching:

\[ \exists \text{ bijection } f: V_1 \rightarrow V_2 \text{ with } u \sim v \text{ in } E_1 \iff f(u) \sim f(v) \text{ in } E_2 \]

Proving nonisomorphism

If some property preserved by isomorphism differs for two graphs, then they're not isomorphic:

- # of nodes,
- # of edges,
- degree distributions, ....
Finding an isomorphism?

many possible mappings: large search
can use properties preserved by isomorphisms as a guide, for example:
• a deg 4 vertex adjacent to a deg 3 can only match with
• a deg 4 vertex also adjacent to a deg 3
but even so...

...nothing known is sure to be much faster than searching thru all bijections for an isomorphism