### Expected Time to Failure

Flip a coin until a Head comes up

\[ \Pr[\text{Head}] = p \]

\[ F ::= \#\text{flips to 1}\text{st Head} \]

**Mean Time to “Failure”**

\[ \Pr[F = 1] = \Pr[H] = p \]

\[ \Pr[F = 2] = \Pr[TH] = q \cdot p \]

\[ \Pr[F = 3] = \Pr[TTH] = q^2 \cdot p \]

PDF of \( F(n) = q^{n-1}p \)

**Geometric Distribution**
Mean Time to “Failure”

\[ E[F] = \sum_{n>0} n \cdot Pr[F=n] \]
\[ = \sum_{n>0} n \cdot q^{n-1}p \]
\[ = p \sum_{n \geq 0} \frac{(n+1)q^n}{(1-q)^2} \]
Mean Time to "Failure"

\[ E[F] = ? \]

\[ p \quad q \quad B \]

\[ H \quad B \]

now use Total Expectation

\[ E[F] = E[F\mid 1^{st} \text{ is } H] \cdot p \]

\[ p \quad q \quad B \]

\[ H \quad B \]

\[ E[F\mid 1^{st} \text{ is } T] \cdot q \]

\[ E[F+1] \]

\[ 1 \]

now solve for \( E[F] \)
Mean Time to “Failure”

\[ E[F] = \frac{1}{1 - p} \]

Mean Time to Failure

application: if space station Mir has \( \frac{1}{150,000} \) chance of destruction in any given hour, how may hours expected until destruction?

\( 150,000 \) hours \( \approx 17 \) years

Intuitive argument

\[ E[\# \text{fails in 1 try}] = p \]
\[ E[\# \text{fails in } n \text{ tries}] = np \]
\[ E[\# \text{tries between fails}] = \frac{\# \text{tries}}{\# \text{fails}} = \frac{n}{np} = \frac{1}{p} \]