Mutually Independent Events

Events $A_1, A_2, \ldots, A_n$ are mutually independent when the probability that $A_i$ occurs is unchanged by which other ones occur.

Example: Successive coin flips
$H_i : [i^{th} \text{ flip is Heads}]$
What happens on the 5th flip is independent of what happens on the 1st, 4th, or 7th flip:

$$\Pr[H_5] = \Pr[H_5 | H_1 \cap H_4 \cap H_7]$$
Mutual Independence

Events $A_1, A_2, \ldots, A_n$ are mutually independent when

$$\Pr[A_i] = \Pr[A_i | A_j \cap A_k \cap \cdots \cap A_m]$$

(i ≠ j, k, ..., m)

Pairwise Independence

Example: Flip a fair coin twice

$H_1 ::= $ [Head on 1st flip]

$H_2 ::= $ [Head on 2nd flip]

$O ::= $ [Odd # Heads]

Claim: $O$ is independent of $H_1$
Not Mutually Independent

Example: Flip a fair coin twice
But $O, H_1, H_2$ not mutually independent:
$$\Pr\left[ O \mid H_1 \cap H_2 \right] = 0 \neq \Pr[O]$$

$k$-way Independence

Events $A_1, A_2, ...$ are $k$-way independent iff any $k$ of them are mutually independent.
Pairwise = 2-way
Mutual Independence

Events $A_1, A_2,\ldots, A_n$ are mutually independent when they are $n$-way independent.

\[
2^n-(n+1) \text{ equations to check!}
\]