Representing Partial Orders

proper subset relation

A ⊂ B means B has everything that A has and more: B ⊄ A

partial order: properly divides

on \{1,2,3,5,10,15,30\}
same shape as $\subset$ example

proper subset

\{1,2,3,5,10,15,30\}

\{1,3,5,15\} \quad \{1,2,5,10\}

\{1,3\} \quad \{1,5\} \quad \{1,2\}

\{1\}

partial order: properly divides

same shape as $\subset$ example isomorphic

on \{1,2,3,5,10,15,30\}
**Isomorphism**

All that matters are the **connections**: graphs with the same connections are **isomorphic**.

**Formal Def of Graph Isomorphism**

\[ G_1 \text{ isomorphic to } G_2 \text{ iff } \exists \text{ bijection } f: V_1 \rightarrow V_2 \text{ with } u \rightarrow v \text{ in } E_1 \iff f(u) \rightarrow f(v) \text{ in } E_2 \]

**p.o. represented by \( \subset \)**

Theorem: Every strict partial order is isomorphic to a collection of subsets partially ordered by \( \subset \).
proof: map element, $a$, to the set of elements below it. $a$ maps to $\{b \in A \mid bRa \text{ OR } b = a\}$

$f(a) ::= R^{-1}(a) \cup \{a\}$