Spanning Subgraphs
A spanning subgraph of graph \( G \) is a subgraph that has all the vertices of \( G \). A spanning tree is a spanning subgraph that is a tree.
Spanning Trees

Lemma: \( G \) connected implies \( G \) has a spanning tree.

Pf: Namely, any minimum edge connected spanning graph.

Minimum Weight Spanning Trees

Suppose edges have weights:

Find min weight spanning tree?
Build MST using gray edges

- Start with vertices
- Color components black & white
- gray edge ::= ○→●
- add min weight gray edge
Minimum Spanning Trees

gray edges: min weight

re-color components

re-color components

gray edges
Minimum Spanning Trees
gray edges: min weight

Minimum Spanning Trees
re-color components

Minimum Spanning Trees
re-color components

Minimum Spanning Trees
etc
Ways to grow an MST

• start at any vertex, keep building one tree. (Prim)
• keep choosing min weight edge between diff components (Kruskal)
• grow trees in parallel (Meyer)