Total Expectation

Law of Total Expectation

good for reasoning by cases

Def: conditional expectation

\[ E[R | A] ::= \sum v \cdot \Pr[R = v | A] \]

\[ E[R] = E[R | A] \cdot \Pr[A] + E[R | A] \cdot \Pr[A] + \ldots \]

More generally, many cases:

Let \( e(n) ::= \) expected #H’s in \( n \) flips.

\[ e(n) = 1 + e(n-1) \quad \text{if 1st flip H} \]
\[ e(n) = e(n-1) \quad \text{if 1st flip T} \]

by Total Expectation:

\[ e(n) = [1 + e(n-1)] \cdot p + e(n-1) \cdot q \]
\[ e(n) = e(n-1) + p = e(n-2) + 2p = \ldots = e(0) + np = np \]
Expected #Heads

Let $e(n) ::= \text{expected #H's in } n \text{ flips.}$

$= 1 + e(n-1)$ if 1st flip H

$= e(n-1)$ if 1st flip T

by Total Expectation:

$$e(n) = (1 + e(n-1))p + e(n-1)q$$

$$e(n) = e(n-1) + p = np = E[B_{n,p}]$$