Introduction to Probability Theory
The Tree Model

Counting in Probability
What is the probability of getting exactly two jacks
in a poker hand?

Counting in Probability
Outcomes: \( \binom{52}{5} \) 5-card hands
Event: \( \binom{4}{2} \binom{52-4}{3} \) hands w/2Jacks
\( \Pr[2 \text{ Jacks}] := \frac{52}{5} \approx 0.04 \)

Probability: 1st Idea
- A set of basic experimental outcomes
- A subset of outcomes is an event
- The probability of an event:
  \( \Pr[\text{event}] := \frac{\# \text{outcomes in event}}{\text{total \# outcomes}} \)
The Monty Hall Game

Applied Probability: Let's Make A Deal (1970's TV Game Show)

Monty Hall Webpages

Image of three doors removed due to copyright restrictions.

http://www.letsmakeadeal.com

Monty Hall Webpages

Image of Monty Hall and contestants removed due to copyright restrictions.

Monty Carol Merill

http://www.letsmakeadeal.com

The Monty Hall Game
• goats behind two doors
• prize behind third door
• contestant picks a door
• Monty reveals a goat behind an unpicked door
• Contest sticks, or switches to the other unopened door
Analyzing Monty Hall

Marilyn Vos Savant explained Game in magazine -- bombarded by letters (even from PhD’s) debating:

1) sticking & switching equally good
2) switching better

Determine the outcomes. -- using a tree of possible steps can help

Monty Hall SWITCH strategy

SWITCH
Wins: 6
Lose: 6

Monty Hall STICK strategy

Win by sticking iff
Lose by switching

STICK
Lose: 6
Wins: 6
A false conclusion: sticking and switching have same # winning outcomes, so probability of winning is the same for both: 1/2.

Another false argument: after door opening, 1 goat and 1 prize are left. Each door is equally likely to have the prize (by symmetry), so both strategies win with probability: 1/2.
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Monty Hall SWITCH strategy

Pr[switch wins] = $\frac{2}{3}$
Probability: 2nd Idea

Outcomes may have differing probabilities!
Not always uniform.

Finding Probability

Intuition is important but dangerous.
Stick with 4-part method:
1. Identify outcomes
2. Identify event (winning)
3. Assign outcome probabilities
4. Compute event probabilities
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