Mathematics for Computer Science
MIT 6.042J/18.062J

Cardinality
(comparing the size of sets)

Cantor's Idea

A surj B ::= ∃surj func: $A \to B$

think: "$|A| \geq |B|$"

A bij B ::= ∃bijection: $A \to B$

think: "$|A| = |B|$"

pow($\mathbb{N}$) bij ∞-bit-strings

infinite set $\mathbb{N} = \{0,1,2,\ldots\}$

subset: {0, , 2, 3, , , 6, … }

string: 1 0 1 1 0 0 1 …

a bijection from pow($\mathbb{N}$) to *infinite* bit-strings, $\{0,1\}^{\omega}$

Familiar "size" properties

$|A| = |B| = |C|$ IMPLIES $|A| = |C|$

A bij B bij C IMPLIES A bij C

proof:

have bijections $g: A \to B$, $f: B \to C$

need bijection $h: A \to C$

define $h ::= f \circ g$

Familiar "size" properties

$|A| = |B| = |C|$ IMPLIES $|A| = |C|$

A bij B bij C IMPLIES A bij C

$|A| \geq |B| \geq |C|$ IMPLIES $|A| \geq |C|$

A surj B surj C IMPLIES A surj C

Familiar "size" properties

$|A| \geq |B| \geq |A|$ IMPLIES $|A| = |B|$

A surj B surj A IMPLIES A bij B

this is NOT obvious:

Schroeder-Bernstein Thm

UNfamiliar "size" property

"size +1 = size"

for ∞-sizes

Same Size Infinite Sets?

$\{1, 2, 3, 4, 5 \ldots\}$

$\uparrow \uparrow \uparrow \uparrow \uparrow$

$\{0, 1, 2, 3, 4 \ldots\}$

$\mathbb{N}$ "same size" $\mathbb{Z}^+$!

Same Size Infinite Sets?

$$\{0, 1, -1, 2, -2, \ldots\}$$

$$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$$

$$\{0, 1, 2, 3, 4, \ldots\}$$

$\mathbb{N}$ "same size as" $\mathbb{Z}$

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