Derived Variables

A derived variable, $v$, is a function assigning a "value" to each state:

$$v : \text{States} \rightarrow \text{Values}$$

If $\text{Vals} = \mathbb{N}$, say $v$ is "$\mathbb{N}$-valued" or "nonnegative-integer-valued".

Robot on the grid example:
States $= \mathbb{N}^2$. Define the sum-value, $\sigma$, of a state:

$$\sigma(x,y) ::= x+y$$

an $\mathbb{N}$-valued derived variable

Called derived to distinguish from actual variables that appear in a program.
For robot

Actual: $x, y$

Derived: $\sigma$
Derived Variables

Another derived variable:
\[ \pi := \sigma \pmod{2} \]
\( \pi \) is \( \{0,1\} \)-valued

For Fast Exp, have (actual) variable \( Z \).
Proof of termination:
\( Z \) is strictly decreasing & natural number-valued

Termination followed by
Well Ordering Principle:
\( Z \) must take a least value.
then the algorithm is stuck

Strictly Decreasing Variable
Goes down at every step
Weakly Decreasing Variable

Down or constant after each step

Diagonal Robot variables

\( \sigma \): up & down all over the place
neither increasing nor decreasing

\( \pi \): is constant
both weakly increasing & weakly decreasing

(We used to call weakly decreasing variables “nonincreasing” variables.)

OK terminology but remember: nonincreasing is NOT SAME as “not increasing.”
Well ordered sets

Def. A set $W$ of real numbers is well ordered iff it has no infinite decreasing sequence

$$w_0 > w_1 > w_2 > \cdots > w_n > \cdots$$

Termination using WOP on $\mathbb{N}$ generalizes to strictly decreasing variables whose values are in any well ordered set.