

|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

Mathematics for Computer Science  
6.042J/18.062J

# Digital Logic



Albert R Meyer

February 13, 2015

digital.1

|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## Adding in binary

$$\begin{array}{r}
 \phantom{39} \phantom{is} \phantom{100111} \phantom{011100} \\
 \phantom{39} \phantom{is} \phantom{100111} \phantom{011100} \\
 \phantom{39} \phantom{is} \phantom{100111} \phantom{011100} \\
 \hline
 1000011
 \end{array}$$

1 1 1 ← carry



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digital.3

|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## Adding in binary

$$\begin{array}{r}
 39 \text{ is } 100111 \\
 28 \text{ is } 011100 \\
 \hline
 \text{sum} = 67 \text{ is } 1000011
 \end{array}$$



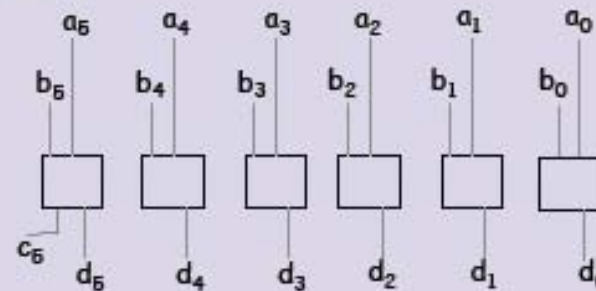
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digital.4

|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## Binary addition circuit



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digital.5

|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## Binary addition circuit

1      0      0      1      1      1

$b_5$   $b_4$   $b_3$   $b_2$   $b_1$   $b_0$

$c_5$   $d_5$   $c_4$   $d_4$   $c_3$   $d_3$   $c_2$   $d_2$   $c_1$   $d_1$   $c_0$   $d_0$

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|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## Binary addition circuit

$a_5$   $a_4$   $a_3$   $a_2$   $a_1$   $a_0$

$b_5$   $b_4$   $b_3$   $b_2$   $b_1$   $b_0$

$c_5$   $d_5$   $c_4$   $d_4$   $c_3$   $d_3$   $c_2$   $d_2$   $c_1$   $d_1$   $c_0$   $d_0$

"ripple carry"

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|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## Binary addition circuit

$a_5$   $a_4$   $a_3$   $a_2$   $a_1$   $a_0$

$b_5$   $b_4$   $b_3$   $b_2$   $b_1$   $b_0$

$c_5$   $d_5$   $c_4$   $d_4$   $c_3$   $d_3$   $c_2$   $d_2$   $c_1$   $d_1$   $c_0$   $d_0$

"ripple carry"

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|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## half Adder

$d ::= a \text{ XOR } b$

$c ::= a \text{ AND } b$

$c$   $d$

from [https://en.wikipedia.org/wiki/Adder\\_\(electronics\)](https://en.wikipedia.org/wiki/Adder_(electronics))

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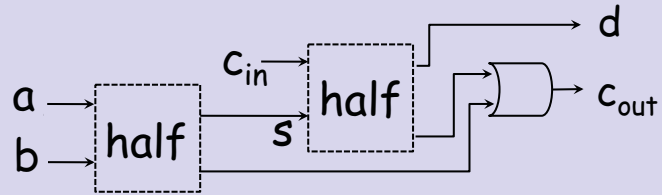
|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## full Adder

$$s ::= a \text{ XOR } b$$

$$d ::= c_{in} \text{ XOR } s$$

$$c_{out} ::= (c_{in} \text{ AND } s) \text{ OR } (a \text{ AND } b)$$



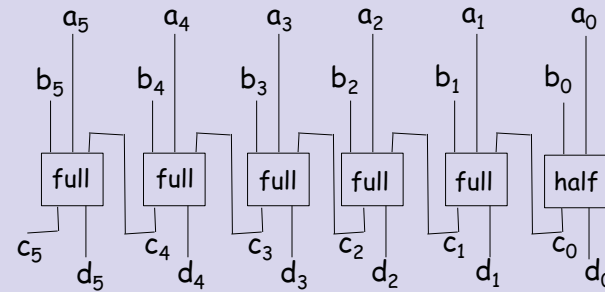
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digital.11

|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## Binary addition circuit



"ripple carry"



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digital.12

|    |   |    |    |
|----|---|----|----|
| 6  | 9 | 13 | 7  |
| 12 |   | 10 | 5  |
| 3  | 1 | 4  | 14 |
| 15 | 8 | 11 | 2  |

## Ripple Carry formulas

$$d_0 ::= a_0 \text{ XOR } b_0$$

$$c_0 ::= a_0 \text{ AND } b_0$$

$$s_i ::= a_i \text{ XOR } b_i$$

$$d_i ::= c_{i-1} \text{ XOR } s_i$$

$$c_i ::= (c_{i-1} \text{ AND } s_i) \text{ OR } (a_i \text{ AND } b_i)$$



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