Finite Cardinality

Mapping Rule (bij)

A bijection from A to B implies
\[ |A| = |B| \]
for finite A, B

Size of the power set

# subsets of a finite set A?
|pow(A)| ?
for A = \{a, b, c\}, pow(A) =
{ \emptyset, \{a\}, \{b\}, \{c\},
{a,b}, \{a,c\}, \{b,c\}, \{a,b,c\} }

pow(A) bijection to bit-strings

A: \{a_0, a_1, a_2, a_3, a_4, \ldots, a_{n-1}\}
subset: \{a_0, a_2, a_3, \ldots, a_{n-1}\}
string: 1 0 1 1 0 \ldots 1
this defines a bijection, so
# n-bit strings = |pow(A)|
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pow(A) bijection to bit-strings

every computer scientist knows $\#n$-bit strings, so
Corollary:

$|\text{pow}(A)| = 2^n$

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function & surjective

$\leq 1$ arrow out $\geq 1$ arrow in

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Mapping Rule (surj)

function: $A \rightarrow B$

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Mapping Rule (surj)

$[\leq 1 \text{out}]: A \rightarrow B$

implies $|A| \geq \#\text{arrows}$.

surjection: $A \rightarrow B$
Mapping Rule (surj)

\[\leq 1 \text{ out}]: A \rightarrow B\]
IMPLIES \(|A| \geq \#\text{arrows} \).

\[\geq 1 \text{ in}]: A \rightarrow B\]
IMPLIES \#\text{arrows} \geq |B|.

Surjective function from \( A \) to \( B \) implies \(|A| \geq |B|\) for finite \( A, B \).

Mapping Rule (inj)

\[\geq 1 \text{ out} \text{ total}]: A \rightarrow B\]
IMPLIES \(|A| \leq \#\text{arrows} \).

\[\leq 1 \text{ in} \text{ injection}]: A \rightarrow B\]
IMPLIES \#\text{arrows} \leq |B|.
**Mapping Rule (inj)**

Total injective relation from $A$ to $B$ implies $|A| \leq |B|$ for finite $A$, $B$

**“jection” relations**

$A$ bij $B ::= \exists$ bijection:$A \rightarrow B$

$A$ surj $B ::= \exists$ surj func:$A \rightarrow B$

$A$ inj $B ::= \exists$ total inj relation:$A \rightarrow B$

**Mapping Lemma**

$A$ bij $B$ \iff $|A| = |B|$

$A$ surj $B$ \iff $|A| \geq |B|$

$A$ inj $B$ \iff $|A| \leq |B|$ for finite $A$, $B$

**Familiar “size” properties**

$|A| = |B| = |C| \implies |A| = |C|$

$|A| \geq |B| \geq |C| \implies |A| \geq |C|$

$|A| \geq |B| \geq |A| \implies |A| = |B|$

for finite $A$, $B$, $C$
Familiar “size” properties

A bij B bij C IMPLIES A bij C
A surj B surj C IMPLIES A surj C
A surj B surj A IMPLIES A bij B

for finite A, B, C

by the Mapping Lemma

for infinite A, B, C, also

1st two implications: easy

3rd is tricky