Recursive Definitions

Define something in terms of a simpler version of the same thing:

Base case(s) that don't depend on anything else.

Constructor case(s) that depend on simpler cases.

Example Definition: set $E$

Define set $E \subseteq \mathbb{Z}$, recursively:

- **Base case:** $0 \in E$
- **Constructor cases:**
  - If $n \in E$, then
    1. $n + 2 \in E$, if $n \geq 0$;
    2. $-n \in E$, if $n > 0$.

Example Definition: set $E$

1. $n \in E$ and $n \geq 0$, then $n + 2 \in E$:
   
   $0, 0+2, (0+2)+2, ((0+2)+2)+2$
   $0, 2, 4, 6, ...$

2. $n \in E$ and $n > 0$, then $-n \in E$
   
   $-2, -4, -6, ...$
   all even numbers
Recursive Def: Extremal Clause
So, \( E \) contains the even integers
Anything Else? No!

- \( 0 \in E \)
- If \( n \in E \) and \( n \geq 0 \), then \( n+2 \in E \)
- If \( n \in E \) and \( n > 0 \), then \( -n \in E \)

That's All!

Extremal Clause
(Implicit part of definition)

Example Definition: set \( E \)
So \( E \) is exactly the Even Integers

Matched Paren Strings, \( M \)
set of strings, \( M \subseteq \{ [ ], [ ] \}^* \)
- Base: \( \lambda \in M \),
  (the empty string)
- Constructor:
  If \( s, t \in M \), then
  \( [s]t \in M \)
Matched Paren Strings $M$

strings $[s]^+ \in M$

\[
\begin{align*}
[ &] 
  s &= \lambda 
  t &= \lambda \\
[ &][ &] 
  s &= [ ] 
  t &= \lambda \\
[ &][ &][ &] 
  s &= \lambda 
  t &= [ ] \\
[ &][ &] 
  s &= [ ] 
  t &= [ ] \\
[ &][ &][ &][ &] 
  s &= \lambda 
  t &= \lambda \\
\vdots & \vdots & \vdots
\end{align*}
\]

not in $M$

strings starting with $]$ are not in $M$ because

• $\lambda$ does not start with $]$
• $[s]^+$ does not start with $]$

and everything in $M$ arises in one of these two ways

The 18.01 Functions, $F_{18}$

The set $F_{18}$ of functions on $\mathbb{R}$:

$\text{Id}_{\mathbb{R}}$, constant functions, and $\sin x$

are in $F_{18}$.

if $f, g \in F_{18}$, then

• $f + g$, $f \cdot g$, $2^f$,
• the inverse, $f^{(-1)}$, of $f$, and
• $f \circ g$ (the composition of $f$ and $g$)

are in $F_{18}$.

Some functions in $F_{18}$:

$-x = (-1) \cdot x$

$\sqrt{x} = (x^2)^{(-1)}$ --- inverse

$\cos x = (1 - (\sin x \cdot \sin x))^{1/2}$

$\ln x = (2^x \log e)^{(-1)}$