Binary relations
A binary relation associates elements of one set called the domain, with elements of another set called the codomain.

“Registered for” relation $R$

Infix notation:

Jason is registered for 6.042

Notation:

$R(Jason, 6.042) \in R$

$R(Jason) = \text{subjects Jason is registered for}$

Images under $R$

$R(Jason) = \text{subjects Jason is registered for}$
"Registered for" relation $R$

Images under $R$

$R(X) ::= \text{all the subjects being taken by students in the set } X$

Images under $R$

$R(X) ::= \text{everything } R \text{ relates to things in } X$

Images under $R$

$R(\{\text{Jason, Yihui}\}) = \text{subjects with Jason or Yihui registered}$
Images under $R$

$R(\{\text{Jason, Yihui}\}) = \{6.042, 6.012, 6.004\}$

“registers” relation $R^{-1}$

```
\begin{array}{c}
\text{student} \\
\text{Jason} \\
\text{Joan} \\
\text{Yihui} \\
\text{Adam} \\
\end{array}
```

```
\begin{array}{c}
\text{subject} \\
\text{registered for} \\
6.042 \\
6.003 \\
6.012 \\
6.004 \\
\end{array}
```

$R^{-1}(6.012) = \{6.042, 6.012, 6.004\}$

$R^{-1}$ relation

```
\begin{array}{c}
\text{student} \\
\text{Jason} \\
\text{Joan} \\
\text{Yihui} \\
\text{Adam} \\
\end{array}
```

```
\begin{array}{c}
\text{subject} \\
\text{registered} \\
6.042 \\
6.003 \\
6.012 \\
6.004 \\
\end{array}
```

$d R j \iff j R^{-1} d$
Images under $R^{-1}$

$R^{-1}(6.012) = \{\text{Jason, Yihui}\}$

$R^{-1}(\{6.012, 6.003\}) = \{\text{Jason, Joan, Yihui}\}$

"registers" relation $R^{-1}$

```
<table>
<thead>
<tr>
<th>student</th>
<th>subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jason</td>
<td>6.042</td>
</tr>
<tr>
<td>Joan</td>
<td>6.003</td>
</tr>
<tr>
<td>Yihui</td>
<td>6.012</td>
</tr>
<tr>
<td>Adam</td>
<td>6.004</td>
</tr>
</tbody>
</table>
```

Inverse image under $R$

$R^{-1}(J) =$ all the students registered for some subject

Every student is registered for some subject:

$D \subseteq R^{-1}(J)$

(not true: Adam wasn't registered)

"advises" relation $V$

```
<table>
<thead>
<tr>
<th>professor</th>
<th>student</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARM</td>
<td>Jason</td>
</tr>
<tr>
<td>FTL</td>
<td>Joan</td>
</tr>
<tr>
<td>TLP</td>
<td>Yihui</td>
</tr>
<tr>
<td>LPK</td>
<td>Adam</td>
</tr>
<tr>
<td>PHW</td>
<td></td>
</tr>
</tbody>
</table>
```
Composing $R$ and $V$

$R \circ V \models \text{"prof has advisee registered for"}$

$p(R \circ V)_j \models \text{prof } p \text{ has an advisee registered in subject } j$

$R \circ V$ is the composition of $R$ and $V$
Composing $R$ and $V$

$\text{ARM (R} \circ V\text{)}$ 6.012 because $\text{ARM V Yihui \ AND \ Yihui R}$ 6.012

$p(R \circ V)j$ IFF $\exists d \in D. \ [p V d \ AND \ d R j]$

note: $V, R$ in reverse order

“teaches” relation $T$

Profs should not teach their advisees:

$\forall p \forall j. \ NOT(p(R \circ V)j \ AND \ pT j)$

$T \cap (R \circ V) = \emptyset$

Profs should not teach their advisees:

$\forall p \forall j. \ NOT(p(R \circ V)j \ AND \ pT j)$

$R \circ V \subseteq T$
Binary relations

A binary relation, $R$, from a set $A$ to a set $B$ associates elements of $A$ with elements of $B$.

Binary relation $R$ from $A$ to $B$

**Domain:**
- $a_1$
- $a_2$
- $a_3$

**Codomain:**
- $b_1$
- $b_2$
- $b_3$
- $b_4$

**Graph of $R$:**
- $(a_1, b_2)$
- $(a_1, b_4)$
- $(a_3, b_4)$

**Range of $R$:**
- $\{b_2, b_4\}$

$\text{graph}(R) := \{ (a_1, b_2), (a_1, b_4), (a_3, b_4) \}$
A function, \( F \), from \( A \) to \( B \) is a relation which associates each element, \( a \), of \( A \) with at most one element of \( B \), called \( F(a) \).

Functions are relations

\[
\text{relation } F : A \rightarrow B \text{ is a function IFF } F(a) \leq 1
\]

IFF

\[
[a \in A \text{ AND } a \neq b] \text{ IMPLIES } F(a) = F(b)
\]