Structural Induction

To prove $P(x)$ holds for all $x$ in recursively defined set $R$, prove
• $P(b)$ for each base case $b \in R$
• $P(c(x))$ for each constructor, $c$, assuming ind. hyp. $P(x)$

Matched Paren Strings $M$

Lemma: Every $s$ in $M$ has the same number of $]$’s and [’s.
Proof by structural induction on the definition of $M$
**Matched Paren Strings M**

**Lemma:** Every $s$ in $M$ has the same number of $]$'s and [$'s.

Let $EQ ::= \{\text{strings with same number of } ] \text{ and } [\}$

**Lemma (restated):** $M \subseteq EQ$

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**Structural Induction on $M$**

**Proof:**

**Ind. Hyp.** $P(s) ::= (s \in EQ)$

**Base case ($s = \lambda$):**

$\lambda$ has 0 $]$'s and 0 [$'s, so $P(\lambda)$ is true.

base case is OK

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**Structural Induction on $M$**

**Constructor step:** $s = [r]t$

can assume $P(r)$ and $P(t)$

$\#]$ in $s = \#]$ in $r + \#]$ in $t + 1$

$\#[ $ in $s = \#[ $ in $r + \#[ $ in $t + 1$

so = $\begin{align*}
\text{by } P(r) & \\
\text{by } P(t) & \\
\text{so } P(s) \text{ is true } & \\
\text{construct case is OK} & \\
\end{align*}$

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**Structural Induction on $M$**

so by struct. induct.

$M \subseteq EQ$

QED
Lemma.

\textbf{F18} is \textit{closed} under taking derivatives:
if \( f \in \text{F18} \), then \( f' \in \text{F18} \)

\textit{Class Problem}