Counting in Poker

2 Pair

**Counting 2-pair poker hands**

A 2-pair hand has

- 2 cards of some rank
- 2 cards of a second rank
- 1 card of still a third rank

To count, choose:

- 1\textsuperscript{st} pair rank (13 ranks)
- 2\textsuperscript{nd} pair rank (12 ranks left)
- last card rank (11 ranks left)
counting 2-pair poker hands
then choose:
• 1\textsuperscript{st} pair suits \( \binom{4}{2} \) sets of 2 suits
• 2\textsuperscript{nd} pair suits \( \binom{4}{2} \) sets of 2 suits
• last card suit (4 suits)

does not count 6-tuples correctly

example: choosing K, A, 3, \{\spadesuit, \heartsuit\}, \{\spadesuit, \clubsuit\}, \spadesuit
specifies 2-pair hand:
K\spadesuit, K\heartsuit, A\spadesuit, A\clubsuit, 3\spadesuit

so # 2-pair hands is

\[ 13 \cdot 12 \cdot 11 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4 \]
counting 2-pair poker hands
but the correspondence to 2-pair hands is not a bijection:

\((K, A, 3, \{\heartsuit, \clubsuit\}, \{\diamondsuit, \spadesuit\}, \spadesuit)\)

\(\spadesuit K, \spadesuit K, A \spadesuit, A \heartsuit, \heartsuit 3\)

\((A, K, 3, \{\heartsuit, \spadesuit\}, \{\diamondsuit, \clubsuit\}, \clubsuit)\)

counting 2-pair poker hands
to count, choose:

• 1\textsuperscript{st} pair rank \ (13 ranks)
• 2\textsuperscript{nd} pair rank \ (12 ranks left)
• last card rank \ (11 ranks left)

either pair might be 1\textsuperscript{st}

map from 6-tuples
\((K, A, 3, \{\heartsuit, \clubsuit\}, \{\diamondsuit, \spadesuit\}, \spadesuit)\)
to 2-pair hands

\(\spadesuit K, \spadesuit K, A \spadesuit, A \heartsuit, \heartsuit 3\)

is 2-to-1
counting 2-pair poker hands
so # 2-pair hands is
13 \cdot 12 \cdot 11 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4

NO!

counting 2-pair poker hands
so # 2-pair hands is really
\frac{1}{2} \cdot 13 \cdot 12 \cdot 11 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4