Homework 4

Do the following warmups and problems. Due in class on Friday, 04 Apr 2008.

Open universe: Collaboration, notes, and other sources of information are encouraged. However, avoid looking up answers until you solve the problem (or have tried hard). That policy helps you learn the most from the problems.

Bring a photocopy to class on the due date, trade it for a solution set, and figure out or ask me about any confusing points. Your work will be graded lightly: P (made a reasonable effort), D (did not make a reasonable effort), or F (did not turn in).

Warmups

1. Minimum power
   In lecture we estimated the flight speed that minimizes energy consumption. Call that speed $v_E$. We could also have estimated $v_P$, the speed that minimizes power consumption. What is the ratio $v_P/v_E$?

2. Solitaire
   You start with the numbers 3, 4, and 5. At each move, you choose any two of the three numbers – call the choices $a$ and $b$ – and replace them with $0.8a - 0.6b$ and $0.6a + 0.8b$. The goal is to reach 4, 4, 4. Can you do it? If yes, give a move sequence; if no, show that you cannot.

Problems

3. Bird flight
   a. For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed $v$ depend on mass $M$ and air density $\rho$? In other words, what are the exponents $\alpha$ and $\beta$ in $v \propto \rho^\alpha M^\beta$?

   b. Use that result to write the ratio $v_{747}/v_{\text{godwit}}$ as a product of dimensionless factors, where $v_{747}$ is the minimum-energy speed of a 747, and $v_{\text{godwit}}$ is the minimum-energy speed of a bar-tailed godwit. Then estimate the dimensionless factors and their product. Useful information: $m_{\text{godwit}} \sim 0.4$ kg.

   c. Use $v_{747}$, from experience or from looking it up, to find $v_{\text{godwit}}$. Compare with the speed of the record-setting bar-tailed godwit, which made its 11,570 km journey in 8 days, 12 hours.

4. Hovering and hummingbirds
   A simple model of hovering is that the animal or helicopter (mass $M$ and wingspan $L$) forces air downward to stay aloft.
a. Estimate the downward air speed \( v_{\text{down}} \) needed to hover.

b. Show that the power required to hover is

\[
P \sim \frac{(Mg)^{3/2}}{\rho^{1/2}L}.
\]

c. Estimate \( P \) and \( v_{\text{down}} \) for a person hovering by flapping or waving his or her arms.

d. How does \( P \) depend on \( M \) for geometrically similar animals (same composition and shape but varying size)? In other words, give the exponent \( \beta \) in

\[
P \propto M^\beta.
\]

e. What fraction of its body weight does a hummingbird \((M \sim 3 \text{ g})\) eat every day in order to hover for a working day \((8 \text{ hours})\)? Compare to the fraction for a person in a typical day. [Hummingbirds eat nectar, which is roughly equal parts sugar and water.]

Optional

5. Inertia tensor

[For those who know about inertia tensors.] Here is the inertia tensor (the generalization of moment of inertia) of a particular object, calculated in a lousy coordinate system:

\[
\begin{pmatrix}
4 & 0 & 0 \\
0 & 5 & 4 \\
0 & 4 & 5 \\
\end{pmatrix}
\]

a. Change coordinate systems to a set of principal axes. In other words, write the inertia tensor as

\[
\begin{pmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz} \\
\end{pmatrix}
\]

and give \( I_{xx}, I_{yy}, \) and \( I_{zz} \). \textit{Hint:} What properties of a matrix are invariant when changing coordinate systems?

b. Give an example of an object with a similar inertia tensor. On Friday in class we’ll have a demonstration.

6. Resistive grid
In an infinite grid of 1-ohm resistors, what is the resistance measured across one resistor?

To measure resistance, an ohmmeter injects a current $I$ at one terminal (for simplicity, say $I = 1$ A), removes the same current from the other terminal, and measures the resulting voltage difference $V$ between the terminals. The resistance is $R = \frac{V}{I}$.

*Hint:* Use symmetry. But it’s still a hard problem!