Problem Set 2

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6.690 Introduction to Power Systems

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Reading: Text, Chapter 2 and Section 3 of Chapter 12

Problem 1: Problem 2.7 from the text.

Problem 2: Problem 2.10 from the text.

Problem 3: With reference to the simple R-C circuit shown in Figure 1: \( R = 500\Omega \) and \( C = 40\mu F \).

The switch has been closed for along time before the action begins at time \( t = 0 \).

1. If the switch is opened at time \( t = 0 \), and the current source is a constant \( I = 1A \), what is the capacitor voltage as a function of time?
2. Now change the source to be \( I = \cos \omega t \) with \( \omega = 2\pi \times 60Hz \). Calculate the output voltage as a function of time.
3. For 6.690 Simulate the transients described here using MATLAB and its ODE solver ODE23.

\[ I \]
\[ \rightarrow \]
\[ \downarrow \]
\[ C \]
\[ \uparrow \]
\[ \downarrow \]
\[ R \]

Figure 1: R C Circuit

Problem 4: A 'buck converter' is shown in Figure 2

\[ 20 \text{ mH} \]
\[ \downarrow \]
\[ 4 \Omega \]
\[ \downarrow \]
\[ 50 \]
\[ + \]
\[ \rightarrow \]
\[ - \]
\[ 20 \text{ mH} \]
\[ \downarrow \]
\[ 4 \Omega \]
\[ \downarrow \]
\[ 50 \]
\[ + \]
\[ \rightarrow \]
\[ - \]

Figure 2: Buck Converter

In this circuit, the switch is closed at a rate of 8,000 times per second and opened again after an interval before the next closing. The ratio of closed time to switching interval is known as the 'duty cycle'. We will use the symbol \( D \) for this duty cycle. So for each period the switch is \( ON \) for \( t_{on} = TD \) and then is \( off \) for \( t_{off} = (1 - D)T \). For this problem, \( T = \frac{1}{8,000} \text{ sec.} \)
1. Calculate the average value of the output voltage as a function of the duty cycle. Note that for this type of load, current in the inductor will be continuous (NOT constant! But it will never go to zero).

2. Estimate the peak-peak ripple voltage for a duty cycle of 50%.

3. For 6.690:
   (a) Expand on this last item to estimate the peak-peak ripple voltage as a function of duty cycle.
   (b) Use MATLAB to simulate this thing, assuming a duty cycle of 50% starting from zero current in the inductor. Note you will really be doing two simulations, one for the time the switch is ON and one for the time the switch is OFF, splicing them together using the final state for each period as the initial state for the next period. Does this match your estimates?
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