Problem 1: Load Flow  There is no single 'right' answer to this one, but you should have inserted the right bus incidence matrix, which is this:

```matlab
% This is the node-incidence Matrix
% 1 1 2 3 4 5 6 7 8
NI=[ 1 0 1 0 0 0 0 0; % Bus 1
     -1 -1 0 0 0 0 0 -1; % Bus 2
     0 0 0 0 1 1 0 0; % Bus 3
     0 0 0 0 0 0 -1 0; % Bus 4
     0 1 -1 1 -1 0 0 0; % Bus 5
     0 0 0 -1 0 -1 1 1]; % Bus 6
```

With the real and reactive power loads in the template script, you should have gotten voltage magnitudes like this:

```
Magnitudes

ans =

    0.8950
    0.8903
    1.0000
    1.0000
    0.9126
    0.9188
```

I modified the script to permit selective injection of reactive power thus:

```matlab
% Added Injection of reactive power to try to smooth
% out voltage
Q_i = [0.5 0.25 0 0 0.25 0.1];
Q = [0 -.5 0 0 -.5 0]' + Q_i';
```

The complete script is appended. Anyway, with this set of injections, voltages at the various buses are:
Magnitudes

\[ \text{ans} = \]

1.0102
0.9807
1.0000
1.0000
0.9849
0.9882

You should do a little playing with the script to see what happens.

**Problem 2** Inductance is:

\[ L = \mu_0 N^2 W \left( \frac{x}{g-h} + \frac{D-x}{g} \right) \]

Using co-energy force is:

\[ f^e = \frac{2}{2} \frac{\partial L}{\partial x} = \frac{\mu_0 N^2 I^2 W}{2} \left( \frac{1}{h-g} - \frac{1}{g} \right) \]

**Problem 3** Inductance is:

\[ L = \frac{\mu_0 N^2 WD}{2g} = \frac{4\pi \times 10^{-7} \times 10^4 \times .02 \times .04}{.001} \approx 10\text{mH} \]

The current limit will be:

\[ I_{\text{sat}} = \frac{2gB_{\text{sat}}}{\mu_0 N} = \frac{1.8 \times .001}{100 \times 4\pi 10^{-7}} \approx 14.3\text{A} \]

For the 6.690 part of the problem, we design this to make the saturation current limit be the same as the heating current limit. The ampere-turn limit would then be:

\[ NI = 2HW_wJ = \frac{2B_{\text{sat}}g}{\mu_0} \]

Where \( J = 2 \times 10^6 \) is our current limit. To make the gap equal to:

\[ g = \frac{\mu_0 NI}{2B_{\text{sat}}} = \frac{\mu_0 HW_wJ}{B_{\text{sat}}} = \frac{4\pi \times 10^{-7} \times .02 \times .02 \times 2 \times 10^6}{1.8} \approx .0005585\text{m} \]

Then we pick the number of turns:

\[ N^2 = \frac{2Lg}{\mu_0 WD} = \frac{2L}{\mu_0 WD} \frac{\mu_0 HW_wJ}{B_{\text{sat}}} = \frac{2LJ}{B_{\text{sat}}} \approx \frac{2 \times .01 \times 2 \times 10^6 \times .02 \times .02}{1.8} \approx 1.111 \times 10^4 \]

or \( N = 105 \).

A calculation of inductance yields \( L \approx 9.9176\mu\text{H} \), which is pretty close to our objective. The current limit is:

\[ I_{\text{lim}} = \frac{2 \times .02 \times .02 \times 2 \times 10^6}{105} \approx 15.2\text{A} \]
Problem 4 Rail area is $2 \times 12 \times .1 = 2.4m^2$, and lift force required is $9.812 \times 40,000 \approx 392,480N$, so force density is 163,533 Pa. This is provided by the attractive force: $F = \frac{1}{2}\mu_0 H_g^2$, or

$$H_g = \sqrt{\frac{2F}{\mu_0}} = \sqrt{\frac{163,533}{2\pi \times 10^{-7}}} \approx 5.102 \times 10^5 \text{A/m}$$

This means that $NI = gH_g = 5,102\text{A-T/pole}$.

With constant current, we know $H_g$ will be inversely proportional to gap: $H_g = \frac{NI}{g}$, so lift force density will be inversely proportional to the square of gap:

$$F = F_0 \left(\frac{g_0}{g}\right)^2$$

This implies instability because the slope of the lift force is negative: if gap decreases, force increases, causing a further decrease in gap.

If current is constant, the magnetic coenergy is just magnetic coenergy density times gap volume:

$$W'_m = \frac{1}{2}\mu_0 H_g^2 \times 2Lg (W - x)$$

where $x$ is displacement, assumed positive (to the right). Force would then be

$$f_x = \frac{\partial W'_m}{\partial x} = mu_0 H_g^2 L g = -2 \times 163,533 \times 12 \times .01 \approx 39,248N$$

This force is restoring, as shown in Figure 1.

![Figure 1: Lateral Force vs. Lateral Displacement](image)

Note, however, that to maintain constant lift, the control system will be forced to increase current with lateral displacement, so restoring force would rise with displacement.