Problem Set 2 (due February 18) \(^1\)

Problem 2.1
For each of the statements below decide whether it is true or false. For a true statement, sketch a proof. For a false statement, give a counterexample.

(a) H2 norm of a stable finite order CT LTI state space model is never larger than 100 times its H-Infinity norm.

(b) H-Infinity norm of a stable finite order CT LTI state space model is never larger than 100 times its H2 norm.

(c) H2 norm of a stable finite order DT LTI state space model is never larger than 100 times its H-Infinity norm.

(d) H-Infinity norm of a stable finite order DT LTI state space model is never larger than 100 times its H2 norm.

Problem 2.2
For continuous time (non-LTI) systems \(S_a\) with scalar input \(f = f(t)\) and scalar output \(g = g(t)\), described below, find their L2 gains, as functions of parameter \(a > 0\). Support your answer with arguments (typically, to show that L2 gain of a system equals, say, 0.5, one has to find input/output pairs of infinite energy (i.e. not converging to zero) for which

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the asymptotic (as time converges to infinity) output-to-input energy ratio is arbitrarily close to \(0.25 = 0.5^2\), and, in addition, to show that the asymptotic energy ratio cannot be larger than 0.5.

(a) \(g(t) = a \sin(f(t))\);

(b) \(g(t) = f(at) \sin(t)\);

(c) \(g(t) = af(t) - |f(t - 1)|\).

**Problem 2.3**

Continuous time signal \(q = q(t)\) is the output of a pure double integrator system with input \(f_1 = f_1(t)\), and \(g(t) = q(t) + bf_2(t)\), where \(b > 0\) is a known constant (do the calculations for \(b = 0.1\) and \(b = 10\)). Find an LTI filter \(F = F(s)\) which takes \(g = g(t)\) as an input and outputs an estimate \(\hat{q} = \hat{q}(t)\) of \(q = q(t)\), which is “good” in one of the following interpretations.

(a) Assuming that \(f = [f_1; f_2]\) is white noise, minimize the asymptotic value of the variance of the estimation error \(e = q - \hat{q}\).

(b) Minimize the L2 gain from \(f = [f_1; f_2]\) to the estimation error \(e = q - \hat{q}\) with accuracy 10 percent.

While there are several MATLAB programs available for solving this problem (and it can also be solved analytically), you are asked to find ways to apply `h2syn.m` and `hinfsyn.m` in this setting. To do this, you will have to resolve the “stabilizability” problem: the standard feedback optimization setup requires stabilizability, which does not take place here (as in many other state estimation problems). One way around this is to introduce some (non-optimal) state observer into the picture, and re-write system equations in terms of the state estimation error variables.