Problem Set 7 (due April 7, 2004) \(^1\)

Problem 7.1
Consider the system described by Figure 7.1, where \(P(s) = 1/(s + a)\), and \(a \in \mathbb{R}, \epsilon > 0\) are parameters.

(a) Find analytically (as a function of \(a \in \mathbb{R}\) and \(\epsilon > 0\)) the maximal lower bound 
\[
\gamma = \gamma(a, \epsilon)
\]
of the H-Infinity norm of the transfer matrix from \(w = [w_1; w_2]\) to 
\(z = [z_1; z_2]\), achievable while using a stabilizing LTI controller \(C = C(s)\).

(b) For \(a = 1, \epsilon = 1\) and for all \(\delta > 0\) find analytically (i.e. as a function of \(\delta > 0\)) a 
controller \(C = C_\delta(s)\) which makes the H-Infinity norm of the closed loop transfer 
matrix from \(w\) to \(z\) less than \(\gamma(1, 1) + \delta\).

(c) Check your solution sing MATLAB calculations.

Problem 7.2
Let \(\mathcal{H}_G\) be the Hankel operator associated with the transfer function

\[
G(s) = \frac{1}{s + a} + \frac{1}{s + 2a},
\]

where \(a > 0\) is a positive parameter.

\(^1\)Version of March 31, 2004
Figure 7.1: Diagram for Problem 7.1

(a) Give an analytical expression (as functions of $a > 0$) for all non-zero singular values of $\mathcal{H}_G$, as well as for the corresponding singular vectors.

(b) Give an analytical expression for a Hankel optimal first order reduced model $\hat{G}$ of $G$.

(c) Check your solution using MATLAB calculations.

**Problem 7.3**

Infinite order transfer function $G$ is defined by

$$G(s) = \frac{1}{s - 1} \int_1^2 \frac{da}{s + a}.$$ 

Find a transfer function $\hat{G}$ of a order $m$ (try to make $m$ as small as possible) such that $G - \hat{G}$ is stable, and $\|G - \hat{G}\|_\infty < 0.02$. You are expected to use Hankel optimal model reduction (function `hankmr.m` of MATLAB), combined with approximation of the integral by a finite sum.