Lecture 21

Optimal Routing

Eytan Modiano
Optimal Routing

• View routing as a “global” optimization problem

• Assumptions:
  – The cost of using a link is a function of the flow on that link
  – The total network cost is the sum of the link costs
  – The required traffic rate between each source-destination pair is known in advance
  – Traffic between source-destination pair can be split along multiple paths with infinite precision

• Find the paths (and associated traffic flows) along which to route all of the traffic such that the total cost is minimized
Formulation of optimal routing

- Let $D_{ij} (f_{ij})$ be the cost function for using link $(i,j)$ with flow $f_{ij}$
  - $F_{ij}$ is the total traffic flow along link $(i,j)$
  - $D_{ij}()$ can represent delay or queue size along the link
  - Assume $D_{ij}$ is a differentiable function

- Let $D(F)$ be the total cost for the network with flow vector $F$

- Assume additive cost: $D(F) = \sum_{(ij)} D_{ij} (f_{ij})$

- For S-D pair $w$ with total rate $r_w$
  - $P_w$ is the set of paths between $S$ and $D$
  - $X_p$ is the rate sent along path $p \in P_w$

\[ S.t. \quad \sum_{p \in P_w} X_p = r_w, \quad \forall w \in W \quad \quad f_{ij} = \sum_{all \ p \ containing \ (i,j)} X_p \]
Formulation continued

- Optimal routing problem can now be written as:

\[
\begin{align*}
\min & \quad D(F) \\
\text{subject to} & \quad \sum_{p \in P_w} X_p = r_w, \quad \forall w \in W
\end{align*}
\]

\[
\Rightarrow \quad \min \sum_{(i,j)} D_{(i,j)} \left[ \sum_{p \text{contains } (i,j)} X_p \right] \quad \text{s.t.} \quad \sum_{p \in P_w} X_p = r_w, \quad \forall w \in W
\]
Optimal routing solution

- Let $dD(*)/dx_p$ be the partial derivative of $D$ with respect to $X_p$

- Then,

- $D'_{xp} = dD(*)/dx_p = \sum_{(i,j) \in p} D'_{(i,j)}$
  
  - Where $D'_{(i,j)}$ is evaluated at the total flow corresponding to $x_p$

- $D'_{xp}$ consists of first derivative lengths along path $p$
Optimal routing solution continued

• Suppose now that $X^* = \{x^*_p\}$ is an optimal flow vector for some S-D pair $w$ with paths $P_w$

• Any shift in traffic from any path $p$ to some other path $p'$ cannot possibly decrease the total cost (since $X^*$ is assumed optimal)

• Define $\Delta$ as the change in cost due to a shift of a small amount of traffic ($\delta$) from some path $p$ with $x^*_p > 0$ to another path $p'$

$$\Delta = \delta \frac{\partial D(X^*)}{\partial x_p} - \delta \frac{\partial D(X^*)}{\partial x_{p'}} \geq 0 \Rightarrow \frac{\partial D(X^*)}{\partial x_{p'}} \geq \frac{\partial D(X^*)}{\partial x_p}, \forall p' \in P_w$$

• Optimality conditions (necessary and sufficient):
  - optimal flows can only be positive on paths with minimum first derivative lengths
  - All paths along which $r_w$ is split must have same first derivative lengths
Example
Example, continued
Routing in the Internet

• **Autonomous systems (AS)**
  – Internet is divided into AS’s each under the control of a single authority

• **Routing protocol can be classified in two categories**
  – Interior protocols - operate within an AS
  – Exterior protocols - operate between AS’s

• **Interior protocols**
  – Typically use shortest path algorithms
    Distance vector - based on distributed Bellman-ford
    Link state protocols - Based on “distributed” Dijkstra’s
Distance vector protocols

• Based on distributed Bellman-Ford
   – Nodes exchange routing table information with their neighbors

• Examples:
   – **Routing information protocols (RIP)**
     Metric used is hop-count \( (d_{ij}=1) \)
     Routing information exchanged every 30 seconds

   – **Interior Gateway Routing Protocol (IGRP)**
     CISCO proprietary
     Metric takes load into account
     \( D_{ij} \sim 1/(\mu-\lambda) \) (estimate delay through link)
     Update every 90 seconds
     Multi-path routing capability
Link State Protocols

- Based on Dijkstra’s Shortest path algorithm
  - Avoids loops
  - Routers monitor the state of their outgoing links
  - Routers broadcast the state of their links within the AS
  - Every node knows the status of all links and can calculate all routes using Dijkstra’s algorithm
    - Nonetheless, nodes only send packet to the next node along the route with the packets destination address. The next node will look-up the address in the routing table.

- Example: Open Shortest Path First (OSPF) commonly used in the internet

- Link State protocols typically generate less “control” traffic than Distance-vector
Inter-Domain routing

• Used to route packets across different AS’s

• Options:
  – Static routing - manually configured routes
  – Distance-vector routing
    Exterior Gateway Protocol (EGP)
    Border Gateway Protocol (BGP)

• Issues
  – What cost “metric” to use for Distance-Vector routing
    Policy issues: Network provider A may not want B’s packets routed through its network or two network providers may have an agreement
    Cost issues: Network providers may charge each other for delivery of packets
Bridges, Routers and Gateways

• A Bridge is used to connect multiple LAN segments
  – Layer 2 routing (Ethernet)
  – Does not know IP address
  – Varying levels of sophistication
    Simple bridges just forward packets
    smart bridges start looking like routers

• A Router is used to route connect between different networks using network layer address
  – Within or between Autonomous Systems
  – Using same protocol (e.g., IP, ATM)

• A Gateway connects between networks using different protocols
  – Protocol conversion
  – Address resolution

• These definitions are often mixed and seem to evolve!