When first presented, AC coupling is kind of a mysterious thing. After seeing it enough times, most people come to terms with it in an intuitive way, which is certainly good enough. But it’s useful as an exercise to come to understand it in a way that is more mathematically explicit.

**CLASS EXERCISE:** Consider the following two-source network:

(1) Write a general expression for $V_0$ using superposition.

(2) Approximate $V_0$ in the cases $|z_1| \ll |z_2|$ and $|z_2| \ll |z_1|$.

(Workspace)

Now let’s take a look at AC coupling.
Using superposition, we can write $V_B$ fairly rapidly:

$$V_B = \frac{1}{(R_1 \parallel R_2)C_s + 1} V_A + \frac{R_1 \parallel R_2}{\frac{1}{C_s} + R_1 \parallel R_2} V_S$$

$$= \frac{1}{(R_1 \parallel R_2)C_s + 1} V_A + \frac{(R_1 \parallel R_2)C_s}{(R_1 \parallel R_2)C_s + 1} V_S$$

So the AC coupling setup low-pass filters the supply and high-pass filters the signal source!

For frequencies $\omega \ll \frac{1}{(R_1 \parallel R_2)C}$, $V_B \approx V_A$

For frequencies $\omega \gg \frac{1}{(R_1 \parallel R_2)C}$, $V_B \approx V_S$

This is exactly what we wanted!

For you network theory buffs, we can generalize this development to an arbitrary number of sources:

Notice that if there is one source impedance $z_i$ that is small compared to all of the other source impedances, its corresponding source dominates and $V_0 \approx V_i$. This is why one tends to think of low-impedance voltage sources as “strong.”
Now let’s embark on our study of single-transistor stages. Our overall goal will be to collapse the
details of the stage into three easy-to-remember figures of merit:

- Input impedance
- Gain
- Output impedance

We write things this way to simplify our lives when we go to a cascade of multiple stages:

Overall gain:

\[ a_v \left( \frac{R_l}{R_0 + R_l} \right) a_v = a_v^2 \left( \frac{R_l}{R_0 + R_l} \right) \]

"Interstage Loading"

Note that sometimes it will be convenient to use the Norton equivalents model. We’ll probably do
this when modeling stages within op-amps:

\[ a_v = G_m R_0 \]

As a warm-up, the familiar Common Emitter Stage:
Input Impedance: \( R_i = r_b + r_\pi \)

Output Impedance: \( R_o = R_L \)

Voltage Gain: \[ V_\pi = \frac{r_\pi}{r_b + r_\pi} V_A \rightarrow V_o = g_m R_L \left( \frac{r_\pi}{r_b + r_\pi} \right) V_A \]

\[ a_v = -\left( \frac{r_\pi}{r_b + r_\pi} \right) g_m R_L \]

**Common Base Stage**
First, we can calculate the input impedance using a test voltage source:

\[
i_t = \frac{v_t}{r_b + r_\pi} + g_m \left( \frac{r_\pi}{r_\pi + r_b} \right) v_t
\]

\[
= \frac{1}{r_\pi + r_b} (1 + g_m r_\pi) v_i
\]

But \( g_m r_\pi = \beta \). So

\[
R_{IN} = \frac{v_i}{i_t} = \frac{r_\pi + r_b}{\beta + 1} \approx \frac{1}{g_m}
\]

Output impedance is easy: \( R_0 = R_L \)

And for the gain:

\[
V_A = \frac{r_\pi}{r_b + r_\pi} V_A \rightarrow V_{OUT} = g_m \left( \frac{r_\pi}{r_b + r_\pi} V_A \right) R_L
\]

\[
\frac{V_{OUT}}{V_A} = g_m R_L \left( \frac{r_\pi}{r_b + r_\pi} \right) \approx g_m R_L \left( \frac{r_\pi}{r_\pi} \right) = g_m R_L
\]

So our model for the common base stage becomes

\[
\begin{align*}
V_s & \quad \rightarrow \quad R_s \\
V_A + & \quad \bullet \\
\frac{r_\pi + r_b}{\beta + 1} & \quad \downarrow \\
\downarrow & \quad \downarrow \\
\downarrow & \quad \downarrow
\end{align*}
\]

\[
\begin{align*}
R_L & \quad + \\
\downarrow & \quad \downarrow \\
\downarrow & \quad \downarrow
\end{align*}
\]

\[
\begin{align*}
V_0 & \quad -
\end{align*}
\]
Now notice two things. First, the input impedance to the common base stage is low.

\[ R_{IN} = \frac{r_x + r_b}{\beta + 1} \approx \frac{1}{g_m} \]

Notice that the impact this has on the gain from \( V_S \) to \( V_0 \)

\[ V_0 \approx \frac{1}{r_x + R_S} V_S \cdot g_m R_L \]

\[ \frac{V_0}{V_S} = \frac{1}{1 + g_m R_L} \approx g_m R_L \]

Second, notice how the dependant current source has the effect of “transforming” resistances in the base so that they appear smaller when seen from the emitter:

\[ R_{IN} = \frac{r_x + r_b}{\beta + 1} \]

Summary of common base stage: High gain, low input impedance, high output impedance.
Emitter Follower Stage

Input Impedance: \[ R_I = r_b + r_\pi + (\beta + 1)R_E \]

Resistances in the emitter seem larger when viewed from the base.

Output Impedance: \[ R_0 = R_E \parallel \left( \frac{R_s + r_b + r_\pi}{\beta + 1} \right) \]

Notice how the source resistance effects the \( R_0 \) of this stage.

Voltage Gain: \[ a_v = \frac{(\beta + 1)R_E}{r_b + r_\pi + (\beta + 1)R_E} \approx 1 \]

Summary of Emitter Follower Stage: Unity gain, high input impedance, low output impedance

Emitter followers make good voltage buffers:

Common base stages make good current buffers: