Way back at the beginning of the term, in Recitation 2, we talked about what it means to do engineering design. Quoting from those notes:

“In engineering design, we make use of nature’s laws to build useful machines.”

The Gilbert Principle is a classic example of how we use the exponential $I_C$ vs. $V_{BE}$ relationship to build analog computation elements. Before examining this in more detail, let’s use the class exercise to look at another intriguing case.

**CLASS EXERCISE:** Consider the following

Using KVL, write $V_0$ as a function of $I$, $I_{S1}$, and $I_{S2}$. Can you think of any useful function served by $V_0$?

(Workspace)
How about that?! Note that while the temperature dependence of $V_{BE}$ is complicated, the temperature dependence of $\Delta V_{BE}$ is simple:

$$\Delta V_{BE} = \frac{kT}{q} \ln \left( \frac{I_{C1}}{I_S} \right) - \frac{kT}{q} \ln \left( \frac{I_{C2}}{I_S} \right) = \frac{kT}{q} \ln \left( \frac{I_{C1}}{I_{C2}} \right)$$

If $\frac{I_{C1}}{I_{C2}}$ is independent of temperature, we say that $\Delta V_{BE}$ is “PTAT,” or Proportional to Absolute Temperature.

So…we took advantage of our detailed knowledge of bipolar transistors to build a thermometer. The Gilbert Principle takes advantage of our knowledge in a different way, and for a different end.

Final Note: Do not use circuits like the class exercise without some sort of start-up circuitry. Note that $I=0$ is a valid state.

As we saw in lecture yesterday, the Gilbert Principle is a kind of mathematical shorthand for KVL. Let’s review.

Current square-root circuit:
Write out KVL:
\[ V_{BE1} + V_{BE2} - V_{BE3} - V_{BE4} = 0 \]
\[ V_T \ln \left( \frac{I_1}{I_S} \right) + V_T \ln \left( \frac{I_B}{I_S} \right) - V_T \ln \left( \frac{I_0}{I_S} \right) - V_T \ln \left( \frac{I_0}{I_S} \right) = 0 \]
\[ V_T \ln \left( \frac{I_1 I_B}{I_S^2} \right) = V_T \ln \left( \frac{I_0^2}{I_S^2} \right) \]
\[ I_0 = \sqrt{I_B I_1} = k \sqrt{I_1} \]

Pretty neat. From this example, simple though it is, we can draw a couple of very general conclusions.

1. Translinear circuits are fast.
2. Translinear circuits always involve an even number of \( V_{BE} \)s.

To understand (1), look at all of the \( C_\mu \)s. None of them get Miller multiplied. \( C_\mu_1 \) even gets bootstrapped, while \( C_\mu_4 \) gets shorted out altogether.

To understand (2), consider a tempting but extremely wrong way to implement the square root function.

Translinear principle:
\[ I_1 = I_0^2 \]
\[ I_0 = \sqrt{I_1} \quad \text{(NO!!)} \]

(The units don’t even work.)
KVL:

\[ V_T \ln \left( \frac{I_1}{I_S} \right) = 2V_T \ln \left( \frac{I_0}{I_S} \right) \]

\[ \ln \left( \frac{I_1}{I_S} \right) = \ln \left( \frac{I_0^2}{I_S^2} \right) \]

\[ I_0 = \sqrt{I_S} \sqrt{I_1} \]

Vitally important that all of the \( I_S \)'s cancel out. This is only possible if the number of CW \( V_{BE} \)'s equals the number of CCW \( V_{BE} \)'s.

To finish off the basics, we recall that we sometimes have the freedom to change emitter areas. If we write

\[ I_S = A_E J_S \]

We have

\[ \sum_{CW} V_{BE_{m}} = \sum_{CW} V_{BE_{n}} \]

\[ \prod_{CW} \frac{I_{C_{m}}}{I_{S_{m}}} = \prod_{CW} \frac{I_{C_{n}}}{I_{S_{n}}} \]

\[ \prod_{CW} \frac{I_{C_{m}}}{A_{E_{m}} J_S} = \prod_{CW} \frac{I_{C_{n}}}{A_{E_{n}} J_S} \]

\[ \prod_{CW} \frac{I_{C_{m}}}{A_{E_{m}}} = \prod_{CW} \frac{I_{C_{n}}}{A_{E_{n}}} \]

This is the most general translinear principle.
Now, at last, a Pythagorator. One such cell was shown to you in lecture yesterday. Here is a seven-transistor version.

How to analyze? Start with

\[ I_0 = I_5 + I_6 \]

And

\[ I_7 = I_0 \]

The left Gilbert loop gives:

\[ I_1 I_2 = I_7 I_5 = I_5 I_0 \]

\[ I_x^2 = I_5 I_0 \Rightarrow I_5 = \frac{I_x^2}{I_0} \]

Right Gilbert loop gives:

\[ I_6 = \frac{I_y^2}{I_0} \]

So for the output:

\[ I_0 = I_5 + I_6 = \frac{I_x^2}{I_0} + \frac{I_y^2}{I_0} \]

\[ I_0^2 = I_x^2 + I_y^2 \]

\[ I_0 = \sqrt{I_x^2 + I_y^2} \]