Welcome to the last recitation of 6.301! You clearly deserve to be proud of how far you’ve come and how hard you have worked. Congratulations on your progress.

We’ll finish the term today with completion of the example we started previously, and then we’ll explore a real-world demo to show that we aren’t just making up this charge control stuff.

To refresh your memory, the problem we were working on was the base current drive scenario, calculating how long it takes to get from cutoff to saturation and back again.

\[
I_B = 200\mu A \\
R_L = 1k\Omega \\
V_{CL} = 10.2V \\
\beta_F = 100 \\
\beta_R = 5 \\
\tau_F = 1nS \\
\tau_R = 5nS \\
\tau_S = 29nS
\]

And, the parameter that I forgot to give last time…

\[
\Psi = 0.75V
\]

Now, the complete charge control model is huge:

\[
i_C = \frac{q_F}{\tau_F} - q_R \left( \frac{1}{\tau_R} + \frac{1}{\tau_{BR}} \right) - \frac{d q_R}{d t} - \frac{d q_{VC}}{d t}
\]

\[
i_B = \frac{q_F}{\tau_{BF}} + \frac{d q_F}{d t} + \frac{q_R}{\tau_{OR}} + \frac{d q_R}{d t} + \frac{d q_{VC}}{d t} + \frac{d q_{VE}}{d t}
\]

\[
i_E = -q_F \left( \frac{1}{\tau_F} + \frac{1}{\tau_{BF}} \right) - \frac{d q_F}{d t} + \frac{q_R}{\tau R} - \frac{d q_{VE}}{d t}
\]
Worse, the nonlinearities of the space charge layers make these equations impossible to solve! What do we do? Approximate, of course. Here’s how.

Last time, in order to get from cutoff to the edge of the forward active region, we simply figured out how much charge was needed by the space charge layers. Once we had those:

\[ t = \frac{\Delta q_{VE} + \Delta q_{VC}}{i_B} \]

At this point we don’t know if we will ultimately wind up in saturation or not. The base current is consistent with a forward active collector current of 20mA, but

\[ I_{C(SAT)} = \frac{V_{CL} - V_{CE(SAT)}}{R_L} = \frac{10V}{1k\Omega} = 10mA \]

So the next part of our calculation involves finding out how long it takes to traverse the forward active region. Again because of nonlinearity associated with the space charge layers, we cannot hope to get an exact answer. We can approximate, though:

Time to traverse FAR:

\[ \frac{\Delta q_F + \Delta q_{VC}}{i_B - \frac{i_{C(AVE)}}{\beta_F}} \]

Where

\[ i_{C(AVE)} = \frac{1}{2} \left( 0 + I_{C(SAT)} \right) \]

Take a moment to understand what these equations are saying. We must supply forward charge \( \Delta q_F \), we must charge the base-collector space charge capacitance with \( \Delta q_{VC} \), and the current that we have to accomplish this is minus the base current needed to support reverse injection.

We worked out the time to get to the edge of FAR at 22ns. To cross the far, we must change \( q_F \):

\[ \Delta q_F = \tau_f I_{C(SAT)} = (1nS)(10mA) = 10pC \]
And change the charge on nonlinear capacitor $C_{\mu}$:

\[ \Delta q_{VC} = q_{VC}(0.4V) - q_{VC}(-9.6V) = 17.6 \text{ pC} \]

During the transition, the average collector current is

\[ i_{C(AVE)} = \frac{1}{2}(0 + I_{C(SAT)}) = 5 \text{ mA} \]

So the time to cross the forward active region is

\[ t = \frac{\Delta q_F + \Delta q_{VC}}{i_b - \frac{i_{C(AVE)}}{\beta_F}} = \frac{27.4 \text{ pC}}{0.2 \text{ mA} - 0.05 \text{ mA}} = 183 \text{ ns} \]

Now suppose that we turn the transistor off by reversing the base current, so that it’s now $-0.2 \text{ mA}$. Before we did this, the saturation charge was

\[ (i_B - I_{BO}) = \frac{q_S}{\tau_S} + \frac{d q_S}{dt} \]

\[ q_S = \tau_S (i_B - I_{BO}) = 29 \text{ ns} (0.2 \text{ mA} - 0.1 \text{ mA}) = 2.9 \text{ pC} \]
Now when we reverse the current, we have the following differential equation to solve:

\[ i_B - I_{BO} = -0.3\text{mA} = \frac{q_s}{\tau_s} + \frac{dq_s}{dt} \]

Homogeneous response: \( q_s(t) = k e^{-t/\tau_s} \)

Driven response: \( q_s(t) = (-0.3\text{mA})(29nS) = -8.7\text{pC} \)

Choose k to fit initial conditions: \( ke^{-0/\tau_s} - 8.7\text{pC} = 2.9\text{pC} \)

\[ k = 11.6\text{pC} \]

Total response: \( (11.6\text{pC})e^{-t/\tau_s} - 8.7\text{pC} = q_s(t) \)

To find out when all of the stored charge is zero (=when we leave saturations):

\[ (11.6\text{pC})e^{-t/\tau_s} - 8.7\text{pC} = 0 \]

\[ e^{-t/\tau_s} = \frac{8.7\text{pC}}{11.6\text{pC}} \]

\[ t = \tau_s \ln \left( \frac{11.6\text{pC}}{8.7\text{pC}} \right) = 8.3\text{ns} \]

Time to leave saturation is 8.3ns.

Traversing the forward active region is the same as before…see if you can work it out. The answer this time is that it takes 110ns.