Today we need to wrap up some of the Bode Obstacle Course stuff that we didn't finish on Friday. Before we do, let's start with a class exercise that explores a fundamental trade-off between speed of response and stability.

**CLASS EXERCISE**

Once again, you're asked to control a plant as shown:

\[
\begin{align*}
X(s) & \xrightarrow{\Sigma} \frac{k}{s} \xrightarrow{\frac{1}{10^3s + 1}} Y(s)
\end{align*}
\]

This time, you've decided that what this system needs is a pole at the origin. Choose \( k \) to meet the following requirements:

1) Such that the system has a phase margin of \( \approx 90^\circ \)
2) Such that the system has a phase margin of \( \approx 45^\circ \)

For which \( k \) is the system faster?

This problem illustrates a general property of feedback systems. You'll often hear people say things like, “For reasons related to stability, the bandwidth is limited to \( x \).”
Now, back to Bode Obstacle Course. Let’s return to the example from last time:

We want to design an acceptable $L(s)$ that results in the following closed-loop performance specs:

1) Steady-state error in response to a ramp $< 1$
2) Disturbance rejection better than 10:1 for frequencies below 10 rps.
3) Closed-loop bandwidth $> 50$ rps
4) Magnitude peaking $M_p < 1.4$
5) Noise rejection better than 40 dB above $10^3$ rps

How does this guide our decision?

1) Steady-state error in response to a ramp is bounded, but not zero. This implies one pole @ the origin. Let’s write our loop transmission as

$$ L(s) = \frac{k}{s} F(s) $$

Where $F(s) = \frac{(\tau_{z1}s+1)(\tau_{z2}s+1)\cdots(\tau_{zN}s+1)}{(\tau_{p1}s+1)(\tau_{p2}s+1)\cdots(\tau_{pn}s+1)}$ => $F(0) = 1$

In response to a ramp, steady-state error is

$$ \lim_{s \to 0} s \left( \frac{1}{s^2} \right) \frac{1}{1 + \frac{k}{s} F(s)} = \lim_{s \to 0} \frac{1}{s + kF(0)} = \frac{1}{k} $$
So for our first spec,

\[ \frac{1}{k} < 0.01 \rightarrow k > 100 \]

2) \( \rightarrow |L(j\omega)| > 10 \) for \( \omega < 10 \) rps
3) \( \rightarrow \omega_c > 50 \) rps
4) \( \rightarrow \phi_m > 45^\circ \)
5) \( \rightarrow |L(j\omega)| < 0.01 \) for \( \omega > 10^3 \) rps

A first try, let’s follow sound engineering judgement and with the simplest \( L(s) \) possible:

\[ L(s) = \frac{100}{s} \]
What about a pole right at 100 rps? Using asymptotes on the bode plot, that would fix \( \omega_c \) right at 100 rps, and the phase margin would be 45\(^{\circ}\)....

\[
\text{try } L(s) = \frac{100}{s(0.01s + 1)}
\]

Actual numbers: \( \omega_c \approx 80 \text{ rps}, \phi_m \approx 50^{\circ} \). Success!
Remember:
• Use closed-loop specifications to place constraints on $L(s)$
• Capture as many of those constraints as you can as Bode Obstacles.
• Start simple, and poles/zeros as necessary.

Compensation

The Bode Obstacle course is one tool we have for doing compensation, or “the art of making things better.” In our in-class exercise, we added a pole at the origin and made $k$ as large as we could to make things better. And we noticed that there was a tradeoff between crossover frequency and stability.

So in an ideal world, what would we really want? We would want a magic box that allowed us to set its phase response independent of its magnitude response. For example, we could have arbitrary positive phase shift and a magnitude response of unity for all frequencies.

NATURE DOES NOT ALLOW THIS.

But it allows us something of that flavor. Consider a zero:

$$H(s) = \tau s + 1$$

$$\Delta H(j\omega) = \tan^{-1}(\tau \omega)$$

$$|H(j\omega)| = \sqrt{1 + (\tau \omega)^2}$$

Over the range of frequencies for which $\tau \omega << 1$:

$$\Delta H(j\omega) = \tau \omega$$

$$|H(j\omega)| \approx 1 + \frac{(\tau \omega)^2}{2}$$

the phase increase is more substantial than the magnitude increase! → Zeros can help.