In lecture, Prof. Roberge spoke of building elaborate systems using op-amp circuits. When we do this, we habitually make statements like, “...and we’ll choose the dynamics such that the poles contributed by the op-amps themselves are negligible.” Let’s discuss.

Consider a two-pole system whose behavior interests us only in the range from DC to $\omega_0$. A pole-zero diagram:

The angle $\phi_L$ is small compared to $\phi_S$, $\Rightarrow$ pole @ $\omega_{\text{LARGE}}$ contributes very little phase shift.

For the frequency response evaluated at $s=j\omega_0$:

$$\left. \frac{1}{(\frac{\omega}{\omega_L}+1) (\frac{\omega}{\omega_S}+1)} \right|_{s=j\omega_0} = \frac{1}{(\frac{j\omega_0}{\omega_L}+1) (\frac{j\omega_0}{\omega_S}+1)}$$

$$\frac{\omega_0}{\omega_L} \ll 1$$

$$\frac{\omega_0}{\omega_S}$$ is on the order of unity (at least)!

So this transfer function is well-approximated by the single-pole transfer function $\frac{1}{(\frac{\omega}{\omega_S}+1)}$ for frequencies from DC to $\omega_0$. 

Cite as: Joel Dawson, course materials for 6.302 Feedback Systems, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYYY].
But what determines $\omega_0$? Depends on the context, but for feedback systems we often look at the loop transmission.

![Diagram of a control system](image)

When $|L(s)| = |GH| \ll 1$, we arguably have an open-loop system:

$$\frac{Y(s)}{X(s)} = \frac{G}{1 + GH} \approx G$$

So when we're looking for dynamics to ignore, we will often discard poles that are large compared to the loop crossover frequency, or the frequency at which $|L(S)| = 1$.

**Op-amp circuits for modeling our systems**

We need an integrator, a summer, an inversion, and a gain.

$$V_0 = -\frac{R_f}{R_1} V_i + \left(-\frac{R_f}{R_2}\right) V_2 + \ldots$$
\[ k_j = \frac{R_f}{R_i} \]

(Note that this also covers us for an inversion.)

\[ v_o = \frac{I}{sR_iC} \]

And, a block that we do not use:

\[ \frac{v_o}{v_i} = -sCR \]

Difficult to manage in a noisy world:

\[ |H(s)| \]

High frequency noise is emphasized in the output

\[ 20\text{dB/dec} \]
Also against the differentiator: it’s hard enough to get high gain at DC. High gain at high frequencies? Forget it.

Now, on to building analog computers. Suppose we have an all-pole system. It begins as a differential equation:

\[
a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + a_{n-2} \frac{d^{n-2} y}{dt^{n-2}} + \ldots + a_0 y = x
\]

Completely general procedure starts with taking the Laplace transform:

\[
(a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_0) Y(s) = x(s)
\]

The system function, BTW, is:

\[
\frac{Y(s)}{X(s)} = \frac{1}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_0}
\]

Solve for the highest order derivative:

\[
a_n s^n Y(s) = X(s) - (a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_0) Y(s)
\]

\[
s^n Y(s) = \frac{1}{a_n} X(s) - \left[ \frac{a_{n-1}}{a_n} s^{n-1} + \frac{a_{n-2}}{a_n} s^{n-2} + \ldots + \frac{a_0}{a_n} \right] Y(s)
\]

Put down a big summing junction:

\[
\frac{1}{a_n} X(s) \rightarrow \sum \rightarrow s^n Y(s)
\]
Generate the derivatives that you need:

\[ x(s) \xrightarrow{\frac{1}{a_n}} \Sigma \xrightarrow{s^n Y(s)} \xrightarrow{\frac{1}{s}} s^{n-1} Y(s) \xrightarrow{\frac{1}{s}} \ldots \xrightarrow{\frac{1}{s}} Y(s) \]

Complete the mapping:

\[ x(s) \xrightarrow{\frac{1}{a_n}} \Sigma \xrightarrow{s^n Y(s)} \xrightarrow{\frac{1}{s}} s^{n-1} Y(s) \xrightarrow{\frac{1}{s}} \ldots \xrightarrow{\frac{1}{s}} Y(s) \]

EXAMPLE: First order system

\[ \frac{Y(s)}{x(s)} = \frac{1}{\tau s + 1} \]

\[ (\tau s + 1) Y(s) = x(s) \]

\[ s Y(s) = \frac{1}{\tau} x(s) - \frac{1}{\tau} Y(s) \]

\[ \Rightarrow \]
2nd order system from class: \[
\frac{v_0}{v_i} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}
\]

With a little bit of manipulation, we can write the block diagram as

If we built this as an electronic circuit, it would be analogous to our mechanical system consisting of a mass, a viscous fluid, and a forcing mechanism:
An analog computer might look something like this:

Make sure that $\omega_n$ is small compared to the parasitic poles of the op-amps. Then, we get a very good analog.