Problem 1 Two semi-working street lamps turn on and off independently as follows: within each one-minute interval, a lamp that is on turns off with probability \( p \), and a lamp that is off turns on with probability \( p \). At time \( t = 0, 1, 2 \ldots \) minutes, an observer records the number \( N_t \) of street lamps that are on, as well as the change \( D_t = N_t - N_{t-1} \) from the previous recorded number.

(a) Do \( N_0, N_1, \ldots \) form a Markov process? What is the entropy rate of this sequence?

(b) Do \( D_0, D_1, \ldots \) form a Markov process? What is the entropy rate of this sequence?

Problem 2 Problem 3.6 in Cover and Thomas (first edition), or 3.10 in Cover and Thomas (second edition).

Problem 3 Consider a sequence of IID binary r.v.s \( A_0, A_1, \ldots \) such that \( A_i = 0 \) with probability \( \xi \) and \( A_i = 1 \) with probability \( 1 - \xi \) for some \( 0 < \xi < 1 \). Consider another sequence of IID quaternary r.v.s \( \Xi_0, \Xi_1, \ldots \) such that \( \Xi_i = 0 \) with probability \( \frac{1-\theta}{3} \), \( \Xi_i = 1 \) with probability \( \frac{1-\theta}{3} \), \( \Xi_i = 2 \) with probability \( \frac{1-\theta}{3} \), \( \Xi_i = 3 \) with probability \( \theta \) for some \( 0 < \theta < 1 \). The \( \Xi_i \)s and the \( A_i \)s are all mutually independent. Consider a sequence of quaternary r.v.s \( Z_0, Z_1, \ldots \) such that \( \forall i > 0 \)

\[
Z_i = A_i(\Xi_{i-1} \oplus Z_{i-1}) \oplus \overline{A_i}\Xi_{i-1}
\]

and \( Z_0, \Xi_0 \) are IID, where \( \oplus \) denotes addition mod 4.

(a) What is \( H(Z_i|Z_{i-1}) \)?

(b) What is \( H(Z_i|Z_{i-j}) \)?

(c) Can you find some form of the AEP that holds for the r.v.s \( Z_0, Z_1, \ldots \)?

Problem 4 Problem 4.1 in Cover and Thomas (first or second edition).