Problem 1 Finite length random codes.

Suppose $n$ messages are mapped to codewords selected uniformly at random over all binary strings of length 4, for transmission over the Z-channel:

![Diagram of Z-channel transition probabilities]

The receiver employs maximum likelihood decoding, declaring an error if there are two equally likely messages.

a) For $n = 2$, what is the average probability of error?

b) For $n = 3$, can you find a bound on the average probability of error?

Problem 2

Consider a DMC. Suppose that the decoder performs ML decoding under a mistaken set of transition probabilities. Find conditions on the true and mistaken transition probabilities such that you can provide an error upper bound, which is exponential in code length and which depends on the input PMF, the true transition probabilities and the mistaken ones.