Problem 1- Problem 2.14 from Gallager’s book.

Problem 2- In class, we proved Kraft inequality by mapping each codeword to a rational number in the interval [0,1). In this problem, we want to show it by using its corresponding binary tree. Suppose $C$ is our codebook with $j$ codewords, each with length $l_j$, respectively.

a) When is $C$ prefix free? (express your answer in terms of properties of a corresponding binary tree)

b) Suppose $G$ is a corresponding binary tree of this codebook. Let $M = \max_j(l_j)$. If $G$ was a complete binary tree with depth $M$, how many leaves would it have? How many children does a node in depth $l_j$ have in the $M^{th}$ stage of this tree?

c) By using (a) and (b), try to prove Kraft inequality.

Problem 3- Suppose $X^n$ is a string of $n$ iid binary discrete random symbols $\{X_k : 1 \leq k \leq n\}$, and $n$ is large enough,

a) If $Pr(X_k = 0) = 1/3$ and $Pr(X_k = 1) = 2/3$, what is the entropy of the random variable $X_k$? What fraction of whole sequences with length $n$ are typical? Determine these sequences.

b) If $Pr(X_k = 0) = Pr(X_k = 1) = 1/2$, how many sequences are typical? Find these typical sequences. Intuitively, in each sequence with length $n$, how many ones and zeros do you expect? Do all typical sequences have this property? Explain.