Review of multipath wireless model

The response to $\exp[2\pi ift]$ over $J$ propagation paths with attenuation $\beta_j$ and delay $\tau_j(t)$ is

$$y_f(t) = \sum_{j=1}^{J} \beta_j \exp[2\pi ift - \tau_j(t)]$$

$$= \hat{h}(f, t) \exp[2\pi ift]$$

The response to $x(t) = \int_{-\infty}^{\infty} \hat{x}(f) \exp[2\pi ift]$ is then

$$y(t) = \int_{-\infty}^{\infty} \hat{x}(f) \hat{h}(f, t) \exp(2\pi ift) \, df$$

$$= \int x(t - \tau) h(\tau, t) \, d\tau \quad \text{where}$$

$$h(\tau, t) \longleftrightarrow \hat{h}(f, t); \quad h(\tau, t) = \sum_j \beta_j \delta\{\tau - \tau_j(t)\}$$
How do we define fading for a single frequency input?

\[ y_f(t) = \hat{h}(f, t) \exp[2\pi if t] \]
\[ = |\hat{h}(f, t)| \exp[2\pi if t + i\angle \hat{h}(f, t)] \]
\[ \Re[y_f(t)] = |\hat{h}(f, t)| \cos[2\pi ft + \angle \hat{h}(f, t)] \]

The envelope of this is \(|\hat{h}(f, t)|\), and this is defined as the fading.

\[ \hat{h}(f, t) = \sum_j \beta_j \exp[-2\pi if \tau_j(t)] = \sum_j \exp[2\pi i D_j t - 2\pi if \tau_j^o] \]

This contains frequencies ranging from \(\min D_j\) to \(\max D_j\). Define the Doppler spread of the channel as

\[ D = \max D_j - \min D_j \]

For any frequency \(\Delta\), \(|\hat{h}(f, t)| = |e^{-2\pi i \Delta t} \hat{h}(f, t)|\)
\[ \hat{h}(f, t) = \sum_j \exp\{2\pi i D_j t - 2\pi i f \tau_j^0\} \]

Choose \( \Delta = [\max D_j + \min D]/2 \). Then

\[
\exp(-2\pi i t \Delta) \hat{h}(f, t) = \sum_{j=1}^{J} \beta_j \exp\{2\pi i t (D_j - \Delta) - 2\pi i f \tau_j^0\}
\]

This waveform is baseband limited to \( D/2 \). Its magnitude is the fading. The fading process is the magnitude of a waveform baseband limited to \( D/2 \). The coherence time of the channel is defined as

\[
T_{\text{coh}} = \frac{1}{2D}
\]

\( D \) is linear in \( f \); \( T_{\text{coh}} \) goes as \( 1/f \).
\( \mathcal{D} \) and \( T_{\text{coh}} \) are two of the primary characteristics of a multipath channel. The other two are

\[
\mathcal{L} = \max_j \tau_j(t) - \min \tau_j(t) \quad \text{and} \quad F_{\text{coh}} = \frac{1}{2\mathcal{L}}
\]

\( F_{\text{coh}} \) is the change in carrier frequency required for the fading to change substantially. This is essentially the T/F dual of the relationship between \( T_{\text{coh}} \) and \( \mathcal{D} \).

\[
\exp[2\pi if_{\text{mid}}] \hat{h}(f, t) = \sum_j \beta_j \exp\{ -2\pi if [\tau_j(t) - \tau_{\text{mid}}] \}
\]

The quantity on the right, as a function of \( f \), is “baseband limited” to \( \mathcal{L}/2 \).
\[ \exp[2\pi if \tau_{\text{mid}}] \tilde{h}(f, t) = \sum_j \beta_j \exp\{-2\pi if [\tau_j(t) - \tau_{\text{mid}}]\} \]

The magnitude of the quantity on the left is the fading at \( f \) and \( t \).

The fading in frequency changes significantly over \( 1/4 \) the bandwidth on the right, \( (\mathcal{L}/2) \); \( \mathcal{F}_{\text{coh}} \) is the order of magnitude change in \( f \) over which the fading changes.

The timing recovery at the receiver tends to keep \( \tau_{\text{mid}} \) close to 0.
Let $\Delta = \tilde{f}_c - f_c$ be the frequency offset in demodulation. Let $\hat{g}(f, t) = \hat{h}(f+f_c, t)e^{-2\pi i \Delta t}$.

Then $\hat{g}(f, t)$ is the baseband system function.
Ignoring the noise for now, the response to a complex baseband input $u(t)$ is

$$v(t) = \int_{-W/2}^{W/2} \hat{u}(f) \hat{h}(f+f_c, t) e^{2\pi i (f-\Delta) t} df$$

$$= \int_{-W/2}^{W/2} \hat{u}(f) \hat{g}(f, t) e^{2\pi if t} df$$

By the same relationship between frequency and time we used for bandpass,

$$v(t) = \int_{-\infty}^{\infty} u(t-\tau) g(\tau, t) d\tau$$

where $g(\tau, t)$ is the inverse Fourier transform (for fixed $t$) of $\hat{g}(f, t)$. 
For the simplified multipath multipath model, \( \hat{h}(f, t) = \sum_{j=1}^{J} \beta_j \exp\{-2\pi if\tau_j(t)\} \) and thus the baseband system function is

\[
\hat{g}(f, t) = \sum_{j=1}^{J} \beta_j \exp\{-2\pi i(f + f_c)\tau_j(t) - 2\pi i\Delta t\}
\]

We can separate the dependence on \( t \) from that on \( f \) by rewriting this as

\[
\hat{g}(f, t) = \sum_{j=1}^{J} \gamma_j(t) \exp\{-2\pi if\tau_j(t)\} \quad \text{where}
\]

\[
\gamma_j(t) = \beta_j \exp\{-2\pi if_c\tau_j(t) - 2\pi i\Delta t\}
\]
For the ray tracing model, $\hat{h}(f, t) = \sum_j \beta_j(t) \exp\{-2\pi i f \tau_j(t)\}$.

$$\hat{g}(f, t) = \sum_j \beta_j(t) \exp\{-2\pi i (f + f_c) \tau_j(t)\}$$

$g(\tau, t) = \sum_j \beta_j(t) \exp\{-2\pi i f_c \tau_j(t)\} \delta[\tau - \tau_j(t)]$

$v(t) = \sum_j \beta_j(t) \exp\{-2\pi i f_c \tau_j(t)\} u[t - \tau_j(t)]$

In terms of Doppler shifts,

$$v(t) = \sum_j \beta_j(t) \exp\{-2\pi i (f_c \tau_j^0 - D_j t)\} u[t - \tau_j(t)]$$

The recovered carrier $f_c^l$ will be shifted to compensate for systematic Doppler shifts. Thus the shifts relative to the recovered carrier will
lie roughly in the range $\pm D/2$. Thus $T_c = 1/(2D)$. 
Discrete-time baseband model

\[ u(t) = \sum_n u_n \text{sinc}(Wt - n) \]

\[ v(t) = \sum_j \beta_j(t) \exp\{2\pi i f_c \tau_j(t)\} u(t - \tau_j(t)) \]

\[ = \sum_n u_n \sum_j \beta_j(t) \exp\{-2\pi i f_c \tau_j(t)\} \text{sinc}[W(t - \tau_j(t)) - n] \]

The sampled outputs at multiples of \( T = 1/W \), i.e. \( v_m = v(mT) \) are then given by

\[ v_m = \sum_n u_n \sum_j \beta_j(mT) \exp\{-2\pi i f_c \tau_j(mT)\} \text{sinc}[m - n - \tau_j(mT)/T] \]

\[ v_m = \sum_k u_{m-k} \sum_j \beta_j(mT) \exp\{-2\pi i f_c \tau_j(mT)\} \text{sinc}[k - \tau_j(mT)/T] \]

\[ = \sum_k u_{m-k} g_{k,m} \quad \text{where} \quad k = m - n. \]